**Quiz #9** **Math 180** Name:

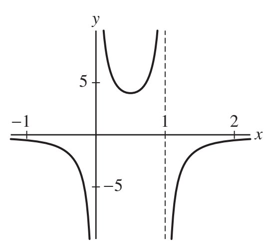
**Problem 1** (6 pts). *Sketch the graph of the function* $f(x) = \frac{1}{x} - \frac{1}{x-1}$. *(Key words:Critical points, increasing, decreasing, first derivative test, inflection points, second derivative test, concave up, concave down, asymptotes)*

**SOLUTION** Let $f(x) = \frac{1}{x} - \frac{1}{x-1}$. Then $f'(x) = \frac{2x-1}{x^2(x-1)^2}$, so that $f$ is decreasing for $x < 0$ and $0 < x < \frac{1}{2}$ and increasing for $\frac{1}{2} < x < 1$ and $x > 1$. Moreover, $f''(x) = -\frac{2\left(3x^2 - 3x + 1\right)}{x^3(x-1)^3}$, so that $f$ is concave up for $0 < x < 1$ and concave down for $x < 0$ and $x > 1$. Because $\lim_{x\to\pm\infty}\left(\frac{1}{x} - \frac{1}{x-1}\right) = 0$, $f$ has a horizontal asymptote at $y = 0$. Finally, $f$ has vertical asymptotes at $x = 0$ and $x = 1$ with

$$\lim_{x\to 0-}\left(\frac{1}{x} - \frac{1}{x-1}\right) = -\infty \quad \text{and} \quad \lim_{x\to 0+}\left(\frac{1}{x} - \frac{1}{x-1}\right) = \infty$$

and

$$\lim_{x\to 1-}\left(\frac{1}{x} - \frac{1}{x-1}\right) = \infty \quad \text{and} \quad \lim_{x\to 1+}\left(\frac{1}{x} - \frac{1}{x-1}\right) = -\infty.$$




**Problem 2** (4pts). . *Find the maximal area among all right triangles with fixed hypotenuse of length* $r$.

**Solution.** *Let* $x$ *and* $y$ *be the length of the two legs of the triangles with fixed hypotenuse of length* $r$. *Then* $x^2 + y^2 = r^2$. *So* $y = \sqrt{r^2 - x^2}$ $x \in [0, r]$. *The area of the triangle is* $A(x) = \frac{1}{2}x\sqrt{r^2 - x^2}$. $A'(x) = \frac{1}{2}(\sqrt{r^2 - x^2} - \frac{x^2}{\sqrt{r^2 - x^2}} = 0 \Longrightarrow x = \frac{\sqrt{2}}{2}r$. $A_{max} = \max A(0), A(\frac{r}{\sqrt{2}}), A(r) = A(\frac{r}{\sqrt{x}}) = \frac{r^2}{4}$