MATH 210 1ST HOUR EXAM FALL 2005

Solution to question 1.
(a) $\vec{AB} = \vec{B} - \vec{A} = (1, 3, -2)$, $\vec{AC} = \vec{C} - \vec{A} = (5, 1, -3)$.The area $= \frac{1}{2}|\vec{AB} \times \vec{AC}|$.

(b) $\vec{BC} = \vec{C} - \vec{B} = (4, -2, -1)$.Since $\vec{AB} \cdot \vec{BC} = 4 - 6 + 2 = 0$. So this is a right triangle.

Solution to question 2.
Since the line on the plane so $(4, 2, 1)$ is a vector parallel to the plane. Pick up another point on the line , say $(-1, 4, 1)$, then there is another vector $(2, -1, 5) - (-1, 4, 1) = (3, -5, 4)$ which is parallel to the plane. So the equation of the plane is

$$\begin{vmatrix} x & y & z \\ 4 & 2 & 1 \\ 3 & -5 & 4 \end{vmatrix}$$

Solution to question 3.
$\vec{v}(t) = \vec{r}'(t) = (1, 2t, 3t^2)$. $v(t) = |\vec{v}(t)| = |(1, 2t, 3t^2)| = \sqrt{1 + 2t^2 + 3t^4}$. $\vec{a}(t) = \vec{v}'(t) = (0, 2, 6t)$.

Solution to question 4. Try it by using Maple. They are circles centered at origin.

Solution to question 5.
The limit does not exist because along the line $y = x$, the limit equals $\frac{1}{2}$ while along the line $y = -x$, the limit equals $-\frac{1}{2}$.

Solution to question 6.
$\frac{\partial f}{\partial y} = -2e^{2x}\sin(2y)$. $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = -4e^{2x}\sin(2y)$. $\frac{\partial^2 f}{\partial y^2} = -4e^{2x}\cos(2y)$.

Solution to question 7.
The point B has the greatest curvature, because the curve is bending sharpest.