MATH 210 SAMPLE EXAM PROBLEMS FOR THE 1ST HOUR EXAM FALL 2009

Solution to question 1.
(a) $\vec{AB} = \vec{B} - \vec{A} = (-1, 0, -1)$, $\vec{AC} = \vec{C} - \vec{A} = (1, 2, -1)$. A point on the plane as a vector can be written as $\vec{A} + s\vec{AB} + t\vec{AC} =$, where s, t are variables.

(b) The area $= \frac{1}{2}|\vec{AB} \times \vec{AC}|$.

Solution to question 2.
Let $\vec{w}$ be the vector $\vec{v} - \vec{u} = (3, 0, -4)$, then the required vector is $\frac{\vec{w}}{|\vec{w}|} = (\frac{3}{5}, 0, -\frac{4}{5})$.

Solution to question 3.
(a) $\vec{T}(0) = \frac{\vec{r}'(0)}{|\vec{r}'(0)|} = \frac{(3,1,0)}{|(3,1,0)|} = (\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}, 0)$.

(b) $|r'(t)| = |(3, e^t, -\sin t)| = \sqrt{9 + e^{2t} + \sin^2 t}$. So the speed of $|r'(t)|$ is the $|r'(t)|' = (\sqrt{9 + e^{2t} + \sin^2 t})' = \frac{2e^{2t} + 2\sin t \cos t}{2\sqrt{9 + e^2 t + \sin^2 t}}$. So the value at $t = 0$ is $\frac{1}{5}$.

Solution to question 4.
(a) As you've seen in 1. the any point on the plane is give by $P + x\vec{u} + y\vec{v}$.

(b) The line is given be $P + t\vec{v}$.

Solution to question 5.
The speed $= |\vec{r}'(t)| = \sqrt{9\sin^2 t + 16\sin^2 t + 25\cos^2 t} = 5$. So the arclength over $[0, 2]$ is $\int_0^2 |\vec{r}'(t)| \mathrm{d}t = 10$.

Solution to question 6.
$\kappa(0) = \frac{|\vec{r}'(0) \times \vec{r}''(0)|}{|\vec{r}'(t)|^3} = \frac{|(1,0,1) \times (1,2,0)|}{|(1,0,1)|^3} =$

Solution to question 7.
(a) easy (b) easy (c) The component is the projection of $\vec{r}''(0)$ onto $\vec{r}'(0)$, it equals to $\frac{\vec{r}''(0) \cdot \vec{r}'(0)}{|vecr'(0)|}$.

Solution to question 8.
Try it by using Maple!

Solution to question 9.
$\dfrac{\partial f}{\partial x} = 2 + 3y$. $\dfrac{\partial f}{\partial y} = 3x - 10y$. $\dfrac{\partial^2 f}{\partial x \partial y} = 3$.

Solution to question 10.
$\dfrac{\partial f}{\partial x} = 2e^{2x}\cos(2y)$. $\dfrac{\partial^2 f}{\partial x^2} = 4e^{2x}\cos(2y)$. $\dfrac{\partial f}{\partial y} = -2e^{2x}\sin(2y)$. $\dfrac{\partial^2 f}{\partial y^2} = -4e^{2x}\cos(2y)$.