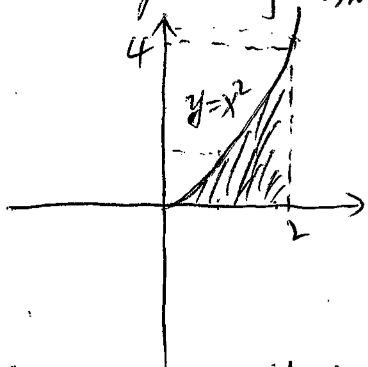
Solutions of Selected Problems

from

Sample Exam Problems for the 2^{nd} Hour Exam Fall 2009

3. The region of integration is $\begin{cases} 0 \le y \le 4 \\ \sqrt{y} \le x \le 2 \end{cases} \iff \begin{cases} 0 \le x \le 2 \\ 0 \le y \le x^2 \end{cases}$



$\Rightarrow$ Change of order

$$\int_0^4 \int_{\sqrt{y}}^2 \sin x^2 \, dx \, dy = \int_0^2 \int_0^{x^2} \sin x^3 \, dy \, dx$$

4. Lagrange Multipliers

Let $g(x,y,z) = \frac{x^2}{4} + \frac{y^2}{9} + z^2$

Then the min and max of $f(x,y,z)$ on $g(x,y,z)=1$ are solutions of the following system of equations

$$\begin{cases} \nabla f = \lambda \nabla g \\ g(x,y,z) = 1 \end{cases}$$

where λ is called the Lagrange Multiplier.

In this case. $\nabla f = \langle 1, 1, -1 \rangle$

$\nabla g = \langle \frac{x}{2}, \frac{2y}{9}, 2z \rangle$

Then

$$\begin{cases} 1 = \lambda \frac{x}{2} \\ 1 = \lambda \frac{2y}{9} \\ -1 = \lambda 2z \\ \frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1 \end{cases} \Rightarrow \lambda = \pm\sqrt{\frac{7}{2}} \Rightarrow \begin{cases} x = \cdots \\ y = \cdots \\ z = \cdots \end{cases}$$

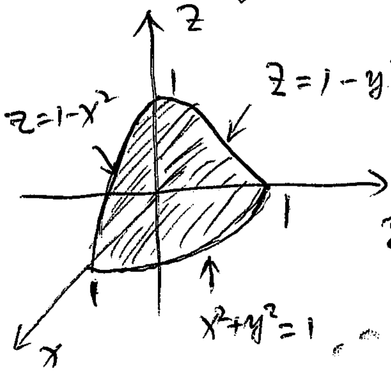
5. The tangent plane to a surface $f(x,y)$ at a point $P = (x_0, y_0, z_0)$ is given by

$$\nabla_P f \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = 0$$

6. see 5

8. see 4

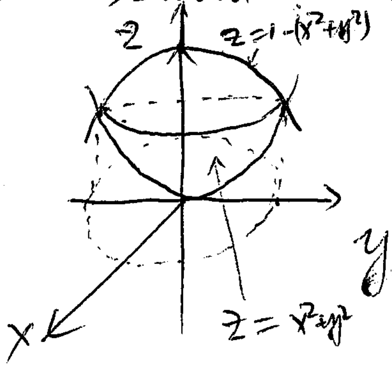
9. $$V = \iint_{\substack{x^2+y^2 \le 1 \\ x \ge 0 \\ y \ge 0}} 1 - (x^2+y^2)\, dx\,dy = \int_0^{\frac{\pi}{2}} \int_0^1 (1 - r^2) \cdot r \, dr\, d\theta$$



10. see 4

11, 12 standard change of variables.

13.



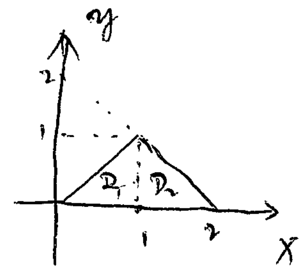
$$\iint_{x^2+y^2 \le 1} 2 - (x^2+y^2) - (x^2+y^2)\, dx\,dy$$

$$= \int_0^{2\pi} \int_0^1 (2 - 2r^2) \cdot r \, dr\, d\theta$$

15

$$\text{Aeverage} = \frac{\iint_A f(x,y)\, dA}{\text{Area of } A} = \frac{\iint_A f(x,y)\, dA}{\iint_A dA}$$

16. $D = D_1 \cup D_2$

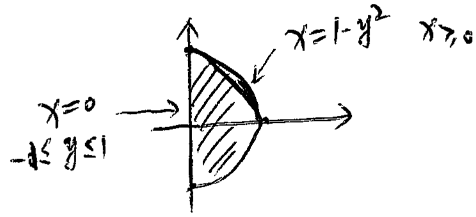


$D_1 = \{ 0 \le x \le 1 ,\ 0 \le y \le x \}$

$D_2 = \{ 1 \le x \le 2 ,\ 0 \le y \le -x+2 \}$

$$\iint_D \frac{x}{y+1}\, dA = \iint_{D_1} \frac{x}{y+1}\, dy\, dx + \iint_{D_2} \frac{x}{y+1}\, dy\, dx$$

17. (b)



(c) Using Lagrange Multiplier to find max min over the boundaries

$$\begin{cases} x = 1-y^2 \\ 0 \le x \end{cases} \quad \text{and} \quad \begin{cases} x=0 \\ -1 \le y \le 1 \end{cases}$$

Then compare the values of $f(x, y)$ at critical points in D with these max and min to get global max and min.

18. $\nabla_{(1,2)} F$

19. $\nabla_{\vec{r}(0)} f \cdot \vec{r}\,'(0)$

20 $\text{mass} = \iiint_A f \, dA$

Using change of variables to compute.