Model Theory and Combinatorics of Homogeneous Metric Spaces

Errata

Gabriel Conant
University of Illinois at Chicago
gconan2@uic.edu

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Typos

- page 20: "...with binary relations given by distance inequalities. However, when working directly..."
- page 25: "...with distances possibly outside of S, still satisfies these axioms."
- page 26: "...there is an $\mathcal{R}$-triangle (r', s', t') in S, which Φ-approximates (u, v, w)."
- page 31: In the proof of Proposition 2.3.10(a), the element s is undefined. The proof should read: "Suppose $\alpha, \beta \in S^*$, with $\alpha \leq^* \beta$. Fix an S-approximation Φ of $\{\alpha, \beta\}$. By density of S, we may fix $r \in \Phi(\alpha) \cap S$ and $s \in \Phi(\beta) \cap S$ such that $r \leq s$. Then (r, s, s) is an $\mathcal{R}$-triangle..."
- page 31: "However, in the case that $\mu := P_S(\alpha, \beta)$ is an element of $\nu(S)$,..."
- page 52: "Call an extension scheme $(\mathcal{A}, f, \Psi)$ **standard** if $\Psi^+(A^f \times A^f) \subseteq R$."
- page 52: "Set $f_0 = f|_{A_0}$, $\mathcal{A}_0 = (A_0, d_A)$, and $\Psi_0 = \Psi_{A_0^{f_0} \times A_0^{f_0}}$."
- page 52: $\Phi(a, b) = \begin{cases} \Phi_0(a, b) & \text{if } a, b \in A_0 \cup \{z_f\} \\ \hat{\Psi}(d_A(a, b)) & \text{otherwise.} \end{cases}$
- page 61: "...and, given $r, s \in R$, $r \oplus s$ is either $r + s$ or the maximal element of R."
- page 63: (from the paragraph starting "In [15]..." through the rest of the section on page 64) In this discussion, there are some technical issues concerning whether equality of theories $T = T'$ should mean that T and T' are the same collection of sentences, or that they axiomatize the

same complete theory. The reader should assume the latter.

- page 64: "In [26], it is shown that the continuous theory of the complete Urysohn sphere has SOP_n for all $n \geq 3$..." (The cited source [26] does not address continuous versions of SOP_1 or SOP_2.)

- page 69: "For example, if $\mathcal{S} = (\{0,1,3,4\}, +_S, \leq, 0)$, then $\frac{1}{2}(1 +_S 3) = 3$ and $\frac{1}{2}1 +_S \frac{1}{2}3 = 4$."

- page 80: "For this, fix $b \in BC$, and note $U(a_2) \leq d(a_2, b) \oplus \delta_b$..."

- page 80: "...by Lemma 3.4.10(c), we have $a' \perp^d_C Bb_*$ for all $a' \in A'$, which gives $A' \perp^d_C Bb_*$ by Lemma 3.4.1."

- page 81: "(iv) $\mathcal{R}$ is ultrametric, i.e., for all $r, s \in R$, if $r \leq s$ then $r \oplus s = s$."

- page 88: "Since $\mathrm{Th}(\mathcal{U}_\mathcal{R})$ is simple, it follows from Theorem 3.5.7(iv)..."

- page 96: "In other words $\mathrm{arch}(\mathcal{R}) \leq n$ if and only if $s \leq nr$ for all $s, r \in R^{>0}$."

- page 98: "Given $1 \leq i < n$, we have $d(a^0_i, a^1_{i+1}) = \alpha_{i+1}$..."

- page 105: "To show the first equality, it suffices by part (a)..."

- page 128: The proof of Theorem 4.4.4 is overly complicated, and contains several typos, listed below. For a cleaner proof, see the preprint *Extending partial isometries of generalized metric spaces*, arXiv 1509.04950.
  - page 128: "Since $\mathrm{Spec}(A)$ is finite, $\mathcal{R}$ is countable and has only finitely many archimedean classes."
  - page 129: The indices i and j should not be fixed at the beginning of the proof of Claim 2. Instead, it should say: *Proof:* We extend A to an $\mathcal{R}$-metric space A^* such that, if $A^*_1, \ldots, A^*_m$ are the $\sim$-classes of A^*, then A^*_i and A^*_j are isometric for all $i, j \leq m$.
  - page 129: "Note that, for all $1 \leq i \leq m$, (A_i, d) is a subspace of (A, d_0)."
  - page 129: "Given $1 \leq i, j \leq p$, fix an isometry $\theta_{i,j} : A_i \longrightarrow A_j$. By induction, there is an $\mathcal{S}_1$-metric space B_1 such that $A_1 \subseteq B_1$ and any partial isometry of A_1 extends to a total isometry of B_1."
  - page 130: In the proof of Claim 4, a_i and a_j are fixed elements of $\mathrm{dom}(\varphi) \cap A_i$ and $\mathrm{dom}(\varphi) \cap A_j$, respectively.
  - page 130: Both the definition of $\hat{\phi}$ and Claim 6 are irrelevant and can be entirely omitted.

- page 148: "(i) $R = \{0, 1, \ldots, n\} \cup \{t\}$, with $t \notin \{0, 1, \ldots, n\}$"

Errors

- page 117: There is a crucial error in the proof of Theorem 4.2.2, which prevents the argument from working in general when $\mathcal{F}$ is nonempty. The argument can be salvaged by imposing strong restrictions on $\mathcal{F}$, but the general situation is unclear. This has the following consequences:

  (i) Theorem 4.2.2 is only proved when $\mathcal{F}$ satisfies certain restrictions (which include $\mathcal{F} = \emptyset$).

  (ii) Corollary 4.2.3 is only proved when $\mathcal{F}$ satisfies these restrictions (in particular, Corollary 4.2.4 is still true).

  (iii) Corollary 4.3.4 is still true (see final remarks below).

  (iv) Other than these, all other results are unaffected.

  A new draft of the argument, which spells out the restrictions on $\mathcal{F}$, is available as part of the preprint: *Extending partial isometries of generalized metric spaces*, `arxiv.org/abs/1509.04950`. In particular, the classes of triangles of odd perimeter (which are the subject of Corollary 4.3.4, and the motivation for considering nonempty $\mathcal{F}$) satisfy these restrictions.