Proposition 1. There are infinitely many primes.

Proof. If there are only finitely many primes then let $N$ be the product of all of them. It follows that a prime factor of $N + 1$ is also a factor of $N$, which is a contradiction. \qed

Definition 2. A Mersenne prime is a prime number of the form $2^n - 1$, where $n$ is an integer.

Definition 3. An integer is perfect if it is equal to the sum of its proper factors.

Theorem 4. An even number is perfect if and only if it is of the form $2^{n-1}(2^n - 1)$, where $2^n - 1$ is a Mersenne prime.

Conjecture 5. There are infinitely many Mersenne primes.

Remark 6. This is one of the most famous open problems in number theory.

Example 7. Only 48 Mersenne prime numbers are known. The largest one is $2^{57885161} - 1$, which is also the largest known prime number.
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