Name______________________________

Let $\mathbf{u}(t) = \left\langle e^{-t^2},\ \ln(t),\ 6 \right\rangle$ and $\mathbf{v}(t) = \left\langle 8 - e^{-t^2},\ -\ln(t),\ \dfrac{2t^3+1}{3t^3} - 6 \right\rangle$.

1. Calculate $\dfrac{d}{dt}\, t\mathbf{u}(t)$.

   Solution
   Using the product rule,

$$\begin{aligned}
\frac{d}{dt}\, t\mathbf{u}(t) &= (t)'\mathbf{u}(t) + t\mathbf{u}'(t) \\
&= \left\langle e^{-t^2},\ \ln(t),\ 6 \right\rangle + t\left\langle -2te^{-t^2},\ \frac{1}{t},\ 0 \right\rangle \\
&= \left\langle e^{-t^2},\ \ln(t),\ 6 \right\rangle + \left\langle -2t^2e^{-t^2},\ 1,\ 0 \right\rangle \\
&= \left\langle (1-2t^2)e^{-t^2},\ \ln(t)+1,\ 6 \right\rangle
\end{aligned}$$

2. Calculate $\lim\limits_{t\to\infty} (\mathbf{u}(t) + \mathbf{v}(t))$.

   Solution
   First,

$$\mathbf{u}(t) + \mathbf{v}(t) = \left\langle 8,\ 0,\ \frac{2t^3+1}{3t^3} \right\rangle$$

   So,

$$\lim_{t\to\infty} (\mathbf{u}(t) + \mathbf{v}(t)) = \lim_{t\to\infty} \left\langle 8,\ 0,\ \frac{2t^3+1}{3t^3} \right\rangle = \left\langle 8,\ 0,\ \frac{2}{3} \right\rangle$$