Name______________________________

Find the length of the trajectory on the given interval.

$$\mathbf{r}(t) = \left\langle e^t \sin t,\ e^t \cos t,\ e^t \right\rangle, \qquad 0 \le t \le \ln 2$$

Solution

Using the product rule,

$$\mathbf{r}'(t) = \left\langle e^t \sin t + e^t \cos t,\ e^t \cos t - e^t \sin t,\ e^t \right\rangle$$

So

$$\begin{aligned}
|\mathbf{r}'(t)| &= \sqrt{(e^t \sin t + e^t \cos t)^2 + (e^t \cos t - e^t \sin t)^2 + (e^t)^2} \\
&= \sqrt{e^{2t} \sin^2 t + 2e^t \sin t \cos t + e^{2t} \cos^2 t + e^{2t} \cos^2 t - 2e^t \cos t \sin t + e^{2t} \sin^2 t + e^{2t}} \\
&= \sqrt{2e^{2t} \sin^2 t + 2e^{2t} \cos^2 t + e^{2t}} \\
&= \sqrt{2e^{2t}(\sin^2 t + \cos^2 t) + e^{2t}} \\
&= \sqrt{3e^{2t}} \\
&= \sqrt{3}e^t
\end{aligned}$$

So the arc length is:

$$\int_0^{\ln 2} \sqrt{3}e^t\ dt = \sqrt{3}e^t \Big|_0^{\ln 2} = \sqrt{3}e^{\ln 2} - \sqrt{3}e^0 = 2\sqrt{3} - \sqrt{3} = \sqrt{3}$$