

Name _____

Calculate

$$\iint_R \frac{e^y}{\sqrt{x}} dA$$

where R is the region bounded by the curves $y = \sqrt{x}$, $y = 0$, $x = 1$, and $x = 4$.**Solution**

The region of integration is bounded above and below by the curves $y = \sqrt{x}$ and $y = 0$; and bounded on the left and right by the lines $x = 1$ and $x = 4$. So the double integral is

$$\int_1^4 \int_0^{\sqrt{x}} \frac{e^y}{\sqrt{x}} dy dx$$

To integrate this:

$$\begin{aligned} \int_1^4 \int_0^{\sqrt{x}} \frac{e^y}{\sqrt{x}} dy dx &= \int_1^4 \left. \frac{e^y}{\sqrt{x}} \right|_0^{\sqrt{x}} dx \\ &= \int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} - \frac{1}{\sqrt{x}} dx \\ &= \int_1^4 \frac{e^{\sqrt{x}} - 1}{\sqrt{x}} dx \quad \left(u = \sqrt{x} \quad du = \frac{1}{2\sqrt{x}} dx \right) \\ &= 2 \int_1^2 e^u - 1 du \\ &= 2e^u - 2u \Big|_1^2 \\ &= 2e^2 - 2e - 2 \end{aligned}$$