

Name \_\_\_\_\_

Compute

$$\int_1^2 \int_0^1 \int_1^{e^y} \frac{e^y}{xz^2} dx dy dz$$

**Solution**

$$\begin{aligned} \int_1^2 \int_0^1 \int_1^{e^y} \frac{e^y}{xz^2} dx dy dz &= \int_1^2 \int_0^1 \frac{e^y \ln x}{z^2} \Big|_1^{e^y} dy dz \\ &= \int_1^2 \int_0^1 \left( \frac{e^y \ln(e^y)}{z^2} - \frac{e^y \ln(1)}{z^2} \right) dy dz \\ &= \int_1^2 \int_0^1 \frac{ye^y}{z^2} dy dz \\ &= \int_1^2 \frac{1}{z^2} \int_0^1 ye^y dy dz \\ &\quad \begin{aligned} u &= y & dv &= e^y dy \\ du &= dy & v &= e^y \end{aligned} \\ &\quad \int ye^y dy = ye^y - \int e^y dy = ye^y - e^y = e^y(y-1) \\ &= \int_1^2 \frac{e^y(y-1)}{z^2} \Big|_0^1 dz \\ &= \int_1^2 \left( \frac{e(1-1)}{z^2} - \frac{e^0(0-1)}{z^2} \right) dz \\ &= \int_1^2 \frac{1}{z^2} dz \\ &= -\frac{1}{z} \Big|_1^2 = -\frac{1}{2} - (-1) = \frac{1}{2} \end{aligned}$$