Proposition 1. *There are infinitely many primes.*

Proof. If there are only finitely many primes then let N be the product of all of them. It follows that a prime factor of $N+1$ is also a factor of N, which is a contradiction. $\square$

Definition 2. A **Mersenne prime** is a prime number of the form $2^n - 1$, where n is an integer.

Definition 3. An integer is **perfect** if it is equal to the sum of its proper factors.

Theorem 4. *An even number is perfect if and only if it is of the form* $2^{n-1}(2^n - 1)$*, where* $2^n - 1$ *is a Mersenne prime.*

Conjecture 5. *There are infinitely many Mersenne primes.*

Remark 6. This is one of the most famous open problems in number theory.

Example 7. Only 48 Mersenne prime numbers are known. The largest one is $2^{57885161} - 1$, which is also the largest known prime number.

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