Sample Proofs

Theorem. *There are infinitely many prime numbers.*

Proof. Suppose, towards a contradiction, that there are not infinitely many prime numbers. Then there are only finitely many, so we can list them as $p_1, p_2, \ldots, p_k$, for some integer $k \geq 1$. Define

$$N = p_1 \cdot p_2 \cdot \ldots \cdot p_k.$$

Then the integer $N+1$ is at least 2, and so must have some prime factor q. By assumption, q divides $N+1$. By definition, q divides N, since it must be one of the primes $p_1, \ldots, p_k$. Therefore q divides $N+1-N=1$. But this is a contradiction, since q is prime and 1 is not divisible by any prime number. □

Theorem. $\ln 2$ *is irrational.*

Proof. Suppose, towards a contradiction, that $\ln 2$ is rational. Then $\ln 2$ can be written as $\frac{m}{n}$, where $m, n \in \mathbb{Z}$, $n \neq 0$, and $m \geq 0$. Note that $m \neq 0$, since otherwise we would have $\ln 2 = 0$, which is not true. So we may assume $m > 0$. Then

$$\begin{aligned} \ln 2 = \frac{m}{n} \quad &\Rightarrow \quad 2 = e^{\frac{m}{n}} \\ &\Rightarrow \quad 2^n = e^m. \end{aligned}$$

Therefore e is a root of the polynomial $x^m - 2^n$, which implies that e is an algebraic number. This is a contradiction, since e is known to be transcendental (not algebraic). □