Math 417 Homework 10

Due November 17

1. Find the order of the zero at the origin of the functions $(e^z - 1)^2$ and $\frac{z^4}{z-\sin(z)}$.

2. Find the order of the poles at the origin of the functions $\frac{1}{(\sin z+z)^3}$ and $\frac{z}{\tan z-\sin z}$.

3. Calculate the contour integrals $\int_{|z-\pi|=1} \frac{z}{\sin(z)}\, dz$ and $\int_{|z-\frac{\pi}{2}|=1} \frac{z^2+1}{\cos(z)}\, dz$.

4. Suppose $f(z)$ is analytic on the unit disc $|z| < 1$ and $f(\frac{1}{n}) = \frac{1}{n^2}$ for each integer $n > 1$. Show that $f(z) = z^2$ for all z.

5. Using contour integration, determine $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+9)}\, dx$.

6. Using contour integration, determine $\int_{-\infty}^{\infty} \frac{x}{x^4+16}\, dx$.