Math 417 Homework 12

Due December 6

1. Find the Mobius transformation taking 1 to 4, 3 to i, and i to $2i$.

2. Find a Mobius transformation that takes the circle $|z-1|=2$ to the line $y=0$. (Hint: One way to do this is to three points on the circle and three points on the line and find the Mobius transformation taking the first three points to the second three points.)

3. Find the image under $f(z)=\frac{1}{z}$ of the set $\{x+iy \in \mathbf{C} : x<1,\, y>0\}$.

4. Let $\alpha \in \mathbf{C}$ with $|\alpha| \neq 1$. Show that if $f(z)=e^{i\theta}\frac{z-\alpha}{1-\bar{\alpha}z}$, then $f(z)$ maps the unit circle $\{z : |z|=1\}$ to itself. Show further that if $|\alpha|<1$ then $f(z)$ maps the interior of this circle to the interior of the circle, while if $|\alpha|>1$ then $f(z)$ maps the interior of the circle to the exterior of the circle.

5. Find a rectangle in the complex plane whose image under $f(z)=e^z$ is the set $\{z : 1<|z|<2,\, \frac{\pi}{2}<\arg(z)<\pi\}$.

6. Show that the image of the line $y=x$ under the exponential map e^z is the spiral of points $z=re^{i\theta}$ satisfying $r=e^{\theta}$.