Math 417 Homework 2
Due September 15

Important Note: Be sure to prove your answer is correct in all problems.

1. Find the domain of $f(z) = \frac{1}{1-i\bar{z}}$ and then write $f(z)$ in $u(x,y) + iv(x,y)$ form.

2. Write the function $f(z) = z^2 - \frac{2}{z^2}$ in the form $u(r,\theta) + iv(r,\theta)$.

3. Sketch the region onto which the sector $\{re^{i\theta} : 0 \leq r \leq 2, 0 \leq \theta \leq \frac{\pi}{4}\}$ is mapped under the transformation $z \to z^2$. Do the same for the transformation $z \to z^3$.

4. Prove that $\lim_{z \to a} \frac{|z|}{z}$ exists at each $a \neq 0$, but does not exist at $a = 0$.

5. Problem 7, page 55 of Churchill and Brown.

6. Prove that $\lim_{z \to 2} \frac{2}{z-2} = \infty$, and that $\lim_{z \to \infty} \frac{z^3+1}{2z^3-3} = \frac{1}{2}$.