Math 417 Homework 3

Due September 22

1. Suppose $f(z)$ is defined on all of $\mathbf{C}$, is differentiable at $z = a$, and satisfies $f(a) = a$. Show that $g(z) = f(f(f(z)))$ is differentiable at $z = a$ and satisfies $g'(a) = (f'(a))^3$.

2. Problem 3, page 61 of Churchill and Brown.

3. Let $f(x+iy) = ix^2$. Using the definition of derivative, similiarly to example 2 on page 57 of Churchill and Brown show that if $Re(a) \neq 0$ then $f(z)$ is not differentiable at $z = a$.

4. Suppose $f(z)$ is defined on $A = \{z : Re(z) > 0\}$ such that $f(z)$ is differentiable at each point in A and such that $(f(z))^2 = z$ for each z in A. Show that for each $z \in A$, $f'(z)$ satisfies $(f'(z))^2 = \frac{1}{4z}$.

5. Let $z = x + iy$. Determine the values of z for which the Cauchy-Riemann equations are satisfied for the following functions.

(a) $f(z) = y^2 + ix^2$
(b) $f(z) = 2x + ixy^2$
(c) $f(z) = xy + ixy$

6. Suppose $f(z) = u(x, y) + iv(x, y)$ is differentiable at $z_0 = x_0 + iy_0$, with $f'(z_0) \neq 0$. Show that the vectors $\nabla u(x_0, y_0)$ and $\nabla v(x_0, y_0)$ are perpendicular and have the same magnitude. Show that the same is true for $g(z) = \bar{z}$ at every point; hence the converse does not hold.