

Math 417 Homework 4

Due September 29

Important Note: Be sure to prove your answer is correct in all problems.

1. Find all harmonic conjugates of $u(x, y) = x^3 - 3xy^2$.
2. Find all (real) values of a, b, c, d such that $ax^3 + bx^2y + cxy^2 + dy^3$ is harmonic on all of \mathbf{C} .
3. Write the following in $a + bi$ form.
 - a) $e^{4 \ln 3 + \frac{\pi}{6} i}$
 - b) $\text{Log}(4 + 4i)$
 - c) $\log(4 + 4i)$
4. Explain why the statement $\log(z^4) = 4 \log(z)$ is false.
5. Explicitly give a branch of $\log z$ that is defined on $\{x + iy : x < 0\}$. Use this to define a branch of $z^{\frac{1}{n}}$ on $\{x + iy : x < 0\}$, where $n > 1$ is a positive integer.
6. In $a + bi$ form find all values of $(3i)^i$. Which is the principal value of $(3i)^i$?