Math 417 Homework 6
Due October 13

1. Let S be the line segment connecting i to 1, oriented either way. Show that $|\int_S \frac{1}{z^2}\,dz| \le 2\sqrt{2}$.

2. Show that

$$\left|\int_{|z|=2} \frac{e^{iz}}{z^3+3}\right| \le \frac{4}{5}\pi e^2$$

3. Show that

$$\lim_{R\to\infty} \int_{|z|=R} \frac{1}{z^2+4z+3}\,dz = 0$$

4. By finding an antiderivative, evaluate each of these integrals, where the path is any contour between the indicated limits of integration:

$$\int_{i-1}^{i+1} z^2\,dz \qquad \int_{-i}^{i} \sin(3z)\,dz \qquad \int_0^{\pi i} e^{2z+\pi i}\,dz$$

5. Problem 5, p. 147 of Churchill and Brown.

6. Using Green's Theorem similarly to as in the proof of a version of the Cauchy-Goursat Theorem from class (and the text), show that if C is a simple closed curve, oriented positively, then the area enclosed by C is given by $\frac{1}{2i}\int_C \bar{z}\,dz$.