Math 417 Homework 7

Due October 27

1. Find the contour integral

$$\int_{|z-1|=1} \frac{e^z}{z^3-1}\, dz$$

2. Let C denote the square with vertices $(-2,-2), (-2,2), (2,-2)$, and $(2,2)$, oriented counterclockwise. Find

$$\int_C \frac{\cos \pi z}{z^2-1}\, dz$$

3. Let D be the circle $|z+1| = 1$, oriented positively. For any positive integer n compute the contour integral

$$\int_D \left(\frac{z+1}{z-1}\right)^n dz$$

Your answer should depend on n.

4. Suppose $\{z_n\}_{n=1}^{\infty}$ and $\{w_n\}_{n=1}^{\infty}$ are sequences of complex numbers such that $\lim_{n\to\infty} z_n = z$ and $\lim_{n\to\infty} w_n = w$ for some complex numbers z and w. Show that for any complex numbers α and β one has $\lim_{n\to\infty} \alpha z_n + \beta w_n = \alpha z + \beta w$.

5. Using the last question, show that if $\{z_n\}_{n=1}^{\infty}$ and $\{w_n\}_{n=1}^{\infty}$ are complex numbers such that $\sum_{n=1}^{\infty} z_n = s$ and $\sum_{n=1}^{\infty} w_n = t$ for some complex numbers s and t, then for any complex numbers α and β one has $\sum_{n=1}^{\infty} \alpha z_n + \beta w_n = \alpha s + \beta t$.

6. Find the Taylor series of $\frac{z}{1+z^2}$ about $z = 0$. Find the region of validity of this Taylor series.