

Math 417 Homework 9

Due November 10

1. Find all isolated singularities of the functions $\frac{1}{z^3-1}$, $\frac{z^2}{1-\cos z}$, and $\frac{z}{\sin(\frac{1}{z})}$.
2. Find all residues of the function $\frac{1}{z(z-2)^4}$. Be sure to include the residue at infinity.
3. Let C denote the circle $|z| = 4$. Use residues to determine $\int_C \frac{1}{(z+3)(z-1)^4} dz$.
4. Using residues at infinity, determine $\int_{|z|=3} \frac{(z+1)^2}{z^2(z-1)} dz$.
5. Let R be the rectangle with vertices $-1 - i, 8 - i, 8 + i, -1 + i$. Determine $\int_R (\frac{2}{z} + \frac{1}{z-3} - \frac{1}{z-7} + \frac{4}{z-9}) dz$.
6. Find $\int_{|z|=1} z^5 e^{\frac{1}{z}} dz$.