EXAM 1 SOLUTIONS

1) By Cauchy Integral formula, $\int_{|z|=2} \frac{e^{\pi i z}}{z-1} = 2\pi i\left(e^{\frac{\pi i \cdot 1}{2}}\right) =$
$2\pi i\left(\cos \pi/2 + i \sin \pi/2\right) = 2\pi i \cdot i = \boxed{-2\pi}$

2) $|2\sqrt{3}+2i| = \sqrt{(2\sqrt{3})^2 + 2^2} = \sqrt{12+4} = 4$. So $2\sqrt{3}+2i = 4e^{i\theta}$ where
$\theta = \tan^{-1}(2/2\sqrt{3}) = \pi/6$. So $2\sqrt{3}+2i = 4e^{i\pi/6}$, $\mathrm{Log}(2\sqrt{3}+2i) =$
$\boxed{\ln 4 + i\pi/6}$, and $\log(2\sqrt{3}+2i) = \boxed{\{\ln 4 + i\pi/6 + 2\pi n i : n \in \mathbb{Z}\}}$

3) On $|z|=2$, $|e^z| = e^{\mathrm{Re}\, z} \le e^2$, $|z^2+2| \geq \left||z^2|-2\right| = 2$, so
by ML rule $\left|\int_{|z|=2} \frac{e^z}{z^2+2}\, dz\right| \le e^2/2 \times 2\pi \times 2 = \boxed{2\pi e^2}$

4) $z(t) = 3t + 2ti : 0 \le t \le 1$, $z'(t) = 3+2i$
$\int_C \bar{z}\, dz = \int_0^1 (3t - 2it)\cdot(3+2i)\, dt = \int_0^1 (3-2i)(3+2i)\, t\, dt$
$= \int_0^1 13t\, dt = \boxed{13/2}$
$\int_C z\, dz = \left(z^2/2 \text{ at } (3+2i)\right) - \left(z^2/2 \text{ at } 0\right)$ (since $z^2/2$ is an antiderivative
for z) $= 1/2\,(3+2i)^2 = \boxed{5/2 + 6i}$

5). $i = e^{i\pi/2}$ from problem 1, so $8i = 8e^{i\pi/2}$. Thus $z^3 = 8e^{i\pi/2}$
implies $z = 2e^{i\pi/6}$, $2e^{i\pi/6 + 2\pi/3\, i}$, or $2e^{i\pi/6 + 4\pi/3\, i}$
So 1): $2e^{i\pi/6} = 2\cos \pi/6 + 2i \sin \pi/6 = \boxed{\sqrt{3} + i}$

2): $2e^{i5\pi/6} = 2\cos 5\pi/6 + 2i \sin 5\pi/6 = \boxed{-\sqrt{3} + i}$
3): $2e^{i9\pi/6} = 2\cos 9\pi/6 + 2i \sin 9\pi/6 = \boxed{-2i}$