Exam 1 Practice Problems

1) Determine the value of $(\sqrt{3}+i)^{42}$ in $a+bi$ form. You may leave whole numbers such as 7^{200}, 5^{80}, etc. in exponential form.

2) Determine all $z_0 \in \mathbf{C}$ such that the function $f(z) = \frac{1}{z^4+16}$ analytic at z_0.

3) Let $g(z)$ be the principal value of $z^{\mathrm{Log}(z)}$, and let A be the domain $\{re^{i\theta} \in \mathbf{C} : r > 0,\ -\pi < \theta < \pi\}$. Explain why $g(z)$ is analytic on A, and find an expression for $g'(z)$.

4) Find the principal values of $(-1)^i$ and $(-1-i)^{-i}$.

5) Show that $u(x,y) = 7x - 2y$ is harmonic at every point of $\mathbf{C}$. Then determine all $v(x,y)$ such that $u(x,y) + iv(x,y)$ is analytic on $\mathbf{C}$.

6) Let C be the circle $|z| = 1.5$. Determine $\int_C \frac{z+1}{z(z+2)}\,dz$ and $\int_C \frac{1}{z^2(z+2)}\,dz$

7) Let C_R denote the circle $|z| = R$. Prove that as long as $R > \sqrt{2}$, one has

$$\left| \int_{C_R} \frac{e^z}{z^2+2i}\,dz \right| \leq 2\pi \frac{Re^R}{R^2-2}$$