

Math 417 Homework 11

Due November 30

1. For any positive integer n , determine the number of solutions to the equation $5z^n = e^z$ lying inside the circle $|z| = 1$.
2. Show that $z^5 + 2z^2 + 3z + 2$ has five zeroes inside the circle $|z| = 2$, and determine the number of zeroes of $z^4 + 7z + 5$ that lie inside the circle $|z| = 1$.
3. Let z_1 and z_2 be any two complex numbers, and let C_1 and C_2 be two distinct circular arcs connecting z_1 and z_2 . Let A be the (bounded) region between C_1 and C_2 . Show that there is some $\theta_0 > 0$ such that there is a linear fractional transformation taking A to a set of the form $\{re^{i\theta} \in \mathbf{C} : -\theta_0 < \theta < \theta_0\}$.
4. Find the image under $f(z) = \frac{1}{z}$ of the set $\{x+iy \in \mathbf{C} : -1 < x < 1, y > 0\}$.
5. Find two circles C_1 and C_2 and a linear fractional transformation T such that C_1 lies inside C_2 and is tangent to C_2 , and such that $T(C_1)$ and $T(C_2)$ are circles which are tangent but neither of $T(C_1)$ nor $T(C_2)$ lies inside the other.