

Math 417 Homework 2

Due September 11

1. Show that $\lim_{z \rightarrow 2} \frac{2}{z-2} = \infty$, and $\lim_{z \rightarrow \infty} \frac{z^3+1}{2z^3-3} = \frac{1}{2}$.
2. Show that any polynomial $a_n z^n + a_{n-1} z^{n-1} + \dots + a_1 z + a_0$ is continuous at every point. The algebraic limit theorem may come in handy for this.
3. Directly from the definitions, show that if $f(z) = \frac{1}{z^2}$ then $f'(z)$ exists and is equal to $-\frac{2}{z^3}$ for all $z \neq 0$. Make sure your argument is rigorous.
4. Show that the image of the rectangle $(0, 1) \times (0, \pi)$ under the function e^z is the portion above the x -axis of the annulus between the circles $|z| = 1$ and $|z| = e$.
5. Show that $\frac{1}{e^{2z}-1}$ is continuous wherever the denominator is nonzero. Then determine the set of all z for which this denominator is nonzero. You may assume from class that e^z is continuous everywhere.