Math 417 Homework 4

Due October 2

1. Show that $\lim_{y\to\infty} \frac{\sin(x+iy)}{e^y} = \frac{ie^{-ix}}{2}$ and $\lim_{y\to\infty} \frac{\cos(x+iy)}{e^y} = \frac{e^{-ix}}{2}$.

2. Suppose we know that $\int_0^{2\pi} e^{(a+bi)t}\,dt = \frac{1}{a+bi}(e^{2\pi(a+bi)} - 1)$, in analogy to what we would get by doing the integral from calculus when $b = 0$. Writing out the real and imaginary parts of both sides of this formula, get formulas for $\int_0^{2\pi} e^{at}\sin(bt)\,dt$ and $\int_0^{2\pi} e^{at}\cos(bt)\,dt$. Do not evaluate these formulas directly using calc II.

3. Let T be the triangle with vertices $(0,0), (2,0)$, and $(0,2)$. Let $f(x+iy) = xy + i(x^2 - 2y)$. Determine $\int_T f(z)$, where one does the integral counterclockwise.

4. Let C be the circle centered at the origin of radius $R > 0$, oriented counterclockwise. For an integer n, show that $\int_C z^n = 0$, unless $n = -1$. Then determine the value of the integral for $n = -1$.