Math 417 Homework 6

Due October 16

1. Let C be the circle $|z| = 2$, oriented positively (counterclockwise). Determine $\int_C \frac{e^{\pi z}}{z^2+1}\, dz$.

2. Let D be the circle $|z+1| = 1$, oriented positively. For any positive integer n compute the contour integral $\int_D \left(\frac{z-1}{z+1}\right)^n dz$. Your answer should depend on n.

3. Let E be the circle $|z| = 1$, oriented positively. For each $r > 0$ other than 1, determine the integral $\int_E \frac{1}{z^2-r}\, dz$.

4. Using Green's Theorem similarly to as in the proof of the version of the Cauchy-Goursat Theorem from class (and the text), show that if C is a simple closed curve, oriented positively, then the area enclosed by C is given by $\frac{1}{2i} \int_C \bar{z}\, dz$.

5. In this problem you will prove a real version of the maximum modulus principle. Let D be the unit disk $\{(x, y) : x^2 + y^2 < 1\}$, and let $\bar{D}$ be the closed disk $\{(x, y) : x^2 + y^2 \leq 1\}$. Let $u(x, y)$ be a continuous function on $\bar{D}$ such that the partials $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$ exist and are continuous on D.

Suppose that the vector ∇u is nonzero at every point (x, y) in D. Then show that if (x_0, y_0) is such that $|u(x_0, y_0)| \geq |u(x, y)|$ for all (x, y) in $\bar{D}$, then (x_0, y_0) is on the boundary of D. In other words, show that one has $x_0^2 + y_0^2 = 1$ and not $x_0^2 + y_0^2 < 1$.