

Math 417 Homework 7

Due October 23

1. For any $z_0 \neq 0$, find the Taylor series of $\frac{1}{z}$ centered around $z = z_0$. Show that this series converges for $|z - z_0| < |z_0|$.
2. Show that if c is any complex number, then $f(z) = ce^z$ satisfies the differential equation $f'(z) = f(z)$ for all z . Conversely, show that if $g(z)$ is any entire function satisfying $g'(z) = g(z)$, then there is some complex number c such that $g(z) = ce^z$. (Hint: examine the Maclaurin expansion of $g(z)$).
3. Find the Laurent series for the function $\frac{1}{z^3} \sin \frac{1}{z^2}$ on $\mathbf{C} - \{0\}$.
4. For any given positive integer n , find the Laurent series for $\frac{1}{z^n(1-z)}$ on the annulus $0 < |z| < 1$ and also the domain $|z| > 1$.
5. Find the Laurent series for $\frac{2}{(z-4)(z-6)}$ on the annulus $4 < |z| < 6$ and also on the domain $|z| > 6$.