Math 417 Homework 9

Due November 13

1. Find the order of the zero at the origin of the functions $(e^z - 1)^2$ and $\frac{z^4}{z-\sin(z)}$.

2. Calculate the contour integrals $\int_{|z-\pi|=1} \frac{e^z}{\sin(z)}\, dz$ and $\int_{|z-\frac{\pi}{2}|=1} \frac{z^2+1}{\cos(z)}\, dz$.

3. Let C be the semicircle consisting of the portion of the circle $|z| = 3$ lying below the x-axis followed by the segment $[-3, 3]$ on the x-axis. Compute $\int_C \frac{z^2}{z^2+4}\, dz$ and $\int_C \frac{1}{(z^2+4)^2}\, dz$.

4. Determine $\int_{|z-3i|=1} \frac{(\mathrm{Log}(z))^2}{z^2+9}\, dz$.

5. Using contour integration, determine $\int_{-\infty}^{\infty} \frac{1}{x^2-4x+5}\, dx$.