

Math 417 Homework 1

Due September 4

Note: In the following problems, you must provide rigorous proofs for your answers. However, you may use without proof anything covered in class.

1. Determine all cube roots of -4 . Express your answers in both polar form and $x + iy$ form.
2. Determine all values of z for which $z^2 - 2iz - (1 + i) = 0$.
3. Determine all values of z for which $|z + i| = |z - i|$.
4. Determine the values of $(1 + \sqrt{3}i)^8$ and $(-1 - i)^{16}$, writing your answers in $x + iy$ form.
5. Suppose $p(z)$ is a polynomial $a_n z^n + a_{n-1} z^{n-1} + \dots + a_1 z + a_0$, such that each of a_0, a_1, \dots, a_n is real. Suppose $x + iy$ is such that $p(x + iy) = 0$. Show that also $p(x - iy) = 0$. If $n = 2$, why can we conclude that either $p(z) = 0$ has two nonreal solutions, or $p(z) = 0$ has no nonreal solutions?