Change of Basis Extra Credit

- Choose among all 10 questions #1-10 from §3.5, p. 161.
- Write up an "**extra-detailed**" (see below) solution for one bonus point each. Everyone is eligible to earn all 10 points. The number of points earned will be ***added to your test score***.

I want solutions to have extra detail for two reasons. First, because all of these answers are in the back of the book. Second, and more importantly, because it is easy to make these calculations without really understanding what is going on. I want you to actually understand it, so here are the requirements.

For each problem, let $E = [\mathbf{e_1}, \mathbf{e_2}, \ldots, \mathbf{e}_n]$ be the standard basis for $\mathbb{R}^n$. (For #4, change the letter E to $\mathcal{B}$.) For any given basis, make a script capital letter that represents it, like $\mathcal{U}$ for basis $[\mathbf{u_1}, \mathbf{u_2}]$. For each question that asks to find the transition matrix S, write **each vector** in terms of its coordinates in the new basis, and **check** that your answer makes sense!

For example, in #3a, the matrix $U^{-1}V = \begin{pmatrix} \frac{5}{2} & \frac{7}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{pmatrix}$.

Congratulations, that's in the back of the book. This answer receives no extra credit. Here's what is required for extra credit, **in addition** to that answer:

$$\mathbf{v_1} = \begin{pmatrix} \frac{5}{2} \\ -\frac{1}{2} \end{pmatrix}_{\mathcal{U}} = \tfrac{5}{2}\mathbf{u_1} + -\tfrac{1}{2}\mathbf{u_2}$$

$$= \tfrac{5}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix}_E + -\tfrac{1}{2}\begin{pmatrix} -1 \\ 1 \end{pmatrix}_E = \begin{pmatrix} 3 \\ 2 \end{pmatrix}_E.$$

It checks. Now do the same for $\mathbf{v_2}$, and #3a will be done.

Due one week from today: Monday the 13th.