Group worksheet: Change of Basis, Row & Column Space

Work in groups. Use your own paper and hand in on Wednesday the 18th. This assignment is worth 20 points, and includes Homework 6. Try to do the problems without referring to your book, but you may use your book for reference. Be sure everyone in your group understands each step of your solution.

1. Find the transition matrix from $[\mathbf{e}_1, \mathbf{e}_2]$ to $[\begin{pmatrix} 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}]$, and determine the coordinates of $\mathbf{x} = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$ in terms of the new basis.

2. Find the transition matrix corresponding to the change of basis from $[\begin{pmatrix} 5 \\ 2 \end{pmatrix}, \begin{pmatrix} 7 \\ 3 \end{pmatrix}]$ to $[\begin{pmatrix} 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}]$.

3. Let $E = [\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3] = [\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 5 \\ 4 \end{pmatrix}]$ and $F = [\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3] = [\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}]$.
   Let $\mathbf{x} = 3\mathbf{v}_1 + 2\mathbf{v}_2 - \mathbf{v}_3$ and $\mathbf{y} = \mathbf{v}_1 - 3\mathbf{v}_2 + 2\mathbf{v}_3$.
   Find the coordinates of $\mathbf{x}$ and $\mathbf{y}$ with respect to F.

4. Find the column space of $A = \begin{pmatrix} 1 & -2 & 1 & 1 & 2 \\ -1 & 3 & 0 & 2 & -2 \\ 0 & 1 & 1 & 3 & 4 \\ 1 & 2 & 5 & 13 & 5 \end{pmatrix}$.

5. Let $A = \begin{pmatrix} 1 & 2 & -1 & 1 \\ 2 & 4 & -3 & 0 \\ 1 & 2 & 1 & 5 \end{pmatrix}$. Find a basis for the row space of A and a basis for $N(A)$. Verify that the nullity equals the number of columns minus the rank.

6. Find the dimension of the subspace of $\mathbb{R}^4$ spanned by
   $\mathbf{x}_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 0 \end{pmatrix}$, $\mathbf{x}_2 = \begin{pmatrix} 2 \\ 5 \\ -3 \\ 2 \end{pmatrix}$, $\mathbf{x}_3 = \begin{pmatrix} 2 \\ 4 \\ -2 \\ 0 \end{pmatrix}$, $\mathbf{x}_4 = \begin{pmatrix} 3 \\ 8 \\ -5 \\ 4 \end{pmatrix}$.

7. §3.6 #1ab.

8. Finish the rest of your homework: §3.5 #3a,4,6
   §3.6 #2ab,3,4cd
   §4.1 #5a