Brief glimpse at some stuff we didn't have time to cover

We know by now that matrices are crucial to the study of linear equations. But they also play an important role in quadratic equations.

Definition

A **quadratic equation** in two variables x and y is an equation of the form

$$ax^2 + 2bxy + cy^2 + dx + ey + f = 0$$

which we can rewrite as

$$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} +$$
$$\begin{pmatrix} d & e \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + f = 0.$$

Let $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$ and $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Then $\mathbf{x}^T A\mathbf{x} = ax^2 + 2bxy + cy^2$.

The term $\mathbf{x}^T A\mathbf{x}$ is called the **quadratic form** associated with the original quadratic equation.

We can use the quadratic form to solve for example:

- Conic sections
- Max/Min optimizations
- Stationary points of curves and surfaces.

Definition

A matrix is **positive definite** $\Leftrightarrow$ its eigenvalues are all positive. These types of matrices occur in many applications:

- Numerical solutions of boundary value problems
- Using finite difference methods
- Using finite element methods.

These are just a couple out of many, many examples. Keep your textbook and use it as a reference for later courses or applications. There are 140 pages we didn't get to!