

HOMework #1  
DUE NOON, JANUARY 16, 2009

1. Prove that  $\sqrt{5}$  is irrational.
2. Prove that for every natural number the inequality  $2n < 3^n$  holds.
3. Define the sequence  $a_1, a_2, a_3, \dots$  in the following way: Let  $a_1 = 1$ . Once  $a_n$  is defined, let  $a_{n+1} = 2a_n + 1$ . Prove that, for each natural number  $n$ , we have  $a_n = 2^n - 1$ .
4. Define the sequence  $b_1, b_2, b_3, \dots$  as follows:  $b_1 = 1$  and  $b_2 = 2$ . Once  $b_{n-1}$  and  $b_n$  are defined, let  $b_{n+1} = 2b_n - b_{n-1}$ .  
Find a formula for  $b_n$ , and prove that your formula is correct.
5. For which values of the real number  $a$  does the equation  $x^2 - 2ax + a = 0$  have at least one real solution  $x$ ?