**Math 313 - Analysis I** **Spring 2009**

# HOMEWORK #4
# DUE NOON, FEBRUARY 6, 2009

(1) Consider the sequences defined by the following formulae. In each case, (a) determine whether the sequence is convergent or divergent; and (b) if the sequence is convergent find the limit.

(i) $a_n = \frac{n}{n+1} - \frac{n+1}{n}$;

(ii) $b_n = \frac{1+(-1)^n}{n}$;

(iii) $c_n = 1 + \frac{n}{n+1} \cos \frac{n\pi}{2}$;

(iv) $d_n = na^n$, for some fixed real number $a$ with $|a| < 1$.

(2) Suppose that $(a_n)$ and $(b_n)$ are Cauchy sequences. Prove that the following sequences are also Cauchy sequences:

(i) $(e_n)$, where $e_n = |a_n - b_n|$;

(ii) $(f_n)$, where $f_n = a_n + b_n$;

(iii) $(g_n)$, where $g_n = a_n b_n$.

(3) Give an example of a pair of Cauchy sequences $(a_n)$, $(b_n)$, where the sequence $(h_n)$ defined by $h_n = \frac{a_n}{b_n}$ is not Cauchy.

(4) Show that the following series are convergent and that the sum is as given:

(i) $\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)} = \frac{1}{2}$;

(ii) $\sum_{n=2}^{\infty} \frac{1}{n^2-1} = \frac{3}{4}$;

(iii) $\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2} = 1$;

(iv) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(2n+1)}{n(n+1)} = 1$.