Math 313 - Analysis I **Spring 2009**

SPRING BREAK PRACTICE PROBLEMS

(1) Suppose that $f : [a, b] \to \mathbb{R}$ is a continuous function so that for all $x \in [a, b]$ we have $f(x) > 0$. Prove that there is some $c > 0$ so that $f(x) \geq c$ for all $x \in [a, b]$. (We say that f is *bounded away from* 0.)

(2) Give an example of a continuous function $f : A \to \mathbb{R}$ (you choose what A is) so that:

(i) f is bounded above but does not attain its maximum;

(ii) f is unbounded;

(iii) f is bounded and attains its maximum but not its minimum.

(3) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(t) = \begin{cases} t - t^3 & \text{if } t \in \mathbb{Q} \\ 0 & \text{if } t \notin \mathbb{Q} \end{cases}$$

Prove that f is continuous at $t = -1, 0, 1$, but not at any other point.

(4) Suppose $f : \mathbb{R} \to \mathbb{R}$ is a function so that for all $x, y \in \mathbb{R}$ we have

$$|f(x) - f(y)| \leq (x - y)^2.$$

Prove that f is a constant function.

(5) Prove that the function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = (1 + \frac{1}{x})^x$$

is increasing when $x > 0$.

(6) Howie, Chapter 3.4, p.80: 3.16 (without using things from later sections, but directly from the definition). Also, 3.17. From 3.5, p.89: 3,21, 3.22, 3.24, 3.26, from 3.6, p. 94: 3.33, from 4.1, pp. 104-5: 4.2, 4.4, from Ch. 4.2, p.110: 4.7, 4.8. Finally, from Ch. 4.3, p.113: 4.13.