

MIDTERM #2 (PRACTICE)  
1PM, APRIL 3, 2009

- (1) Let  $S_7$  be the symmetric group of degree 7, consisting of all permutations of the set  $A = \{1, 2, 3, 4, 5, 6, 7\}$ .

Let  $f : A \rightarrow A$  be defined by the following table:

$x$	1	2	3	4	5	6	7
$f(x)$	2	4	5	1	7	6	3

- (a) Write  $f$  as a product of disjoint cycles.  
(b) What is the order of  $f$ ?  
(c) Write  $f$  as a product of 2-cycles. Is  $f$  even or odd?  
(d) Calculate  $f^{76}$ , written as a product of disjoint cycles.
- (2) Suppose that  $G$  is an abelian group, and  $H \leq G$  is a subgroup.
- (a) Prove that  $H$  is normal in  $G$   
(b) Prove that  $G/H$  is abelian.  
(c) Give an example of a group  $G$  with an abelian normal subgroup  $H$  so that  $G$  is not abelian, but  $G/H$  is abelian.
- (3) (a) Suppose that  $G_1$  is a group and  $g, h \in G_1$ . Prove that if

$$(gh)^2 = g^2h^2$$

then

$$gh = hg.$$

- (b) Suppose that  $G_2$  is a group so that  $g^2 = e$  for all  $g \in G_2$ . Prove that  $G$  is abelian.  
(c) Let  $G_3$  be a group and suppose that all nonidentity elements of  $G_3$  have order 2. Suppose that  $G_3$  has size at least 3. Prove that  $G_3$  has a subgroup of order 4.  
(d) Let  $G_4$  be a group of order 22. Prove that  $G_4$  has an element of order 11.