

SPRING BREAK PRACTICE PROBLEMS

[Do not turn these in for grading. Solutions will be posted on the web during the first week after classes.]

- (1) Suppose that G is a group, $H \leq G$ is a subgroup and $K \trianglelefteq G$ is a normal subgroup. Prove that $H \cap K \trianglelefteq H$.
- (2) Let H be a subgroup of S_n . Let H_0 be the set of H consisting of even permutations.
 - (a) Prove that H_0 is a subgroup of H ;
 - (b) Suppose that $y \in H \setminus H_0$. Prove that $H = H_0 \cup yH_0$. Deduce that either $H = H_0$ or $|H : H_0| = 2$.
 - (c) Prove that $H_0 \trianglelefteq H$.
- (3) Let A be a set and $G \leq \text{Sym}(A)$. Suppose that $x \in A$.
 - (a) Let $g \in G$. Prove that $\text{Stab}_G(g.x) = g\text{Stab}_G(x)g^{-1}$;
 - (b) Suppose that $\text{Stab}_G(x) \trianglelefteq G$. Prove that for all $y \in \text{Orb}_G(x)$ we have $\text{Stab}_G(y) = \text{Stab}_G(x)$.
- (4) Let $A \subseteq \mathbb{R}^2$ be an equilateral triangle centered at the origin, and let $X = A \times [-1, 1]$ be a triangular prism in \mathbb{R}^3 . Let G be the group of rotations in \mathbb{R}^3 which fix the origin and send X to itself (as a set).
 - (a) Let $a = (0, 0, 1)$ be the point in the middle of the 'top' triangle of X . What is the order of the stabiliser of a in G ? What is the size of $\text{Orb}_G(a)$? What is the size of G ?
 - (b) Let $(x, y) \in A$ be one of the corners of A , and consider the point $b = (x, y, 0)$ be the corresponding point in X . What is the size of $\text{Orb}_G(b)$?
 - (c) Show that if $g \in G \setminus \{1\}$ then there is some $y \in \text{Orb}_g(x)$ so that $g.y \neq y$.
 - (d) Find a dihedral group and a symmetric group, each isomorphic to G .
- (5) Prove the Second and Third Isomorphism Theorems (see Gallian, Exercises 39, 40, Chapter 10, p. 213).
- (6) Gallian, Ch.6: # 10, # 12. Ch. 10: # 10, #16, #18.