Example

Find the equations of the tangent lines to the following functions at the given points:

(1) $f(x) = (x-1)e^{x^2}$ at $(1, 0)$.

(2) $f(x) = e^x(x^2 + x + \ln(x))$ at $(1, 2e)$.

(3) $f(x) = \ln(x^2)\frac{x+2}{2x-2}$ at $(2, 2\ln(4))$.

Example:

Suppose that the cost function of producing x thousand units of stuff (in dollars) is:

$$C(x) = 1000(x^2 + 20xe^{-0.1x}).$$

(1) Find the marginal cost $C'(x)$.

(2) For what value of x is the average cost $\frac{C(x)}{x}$ minimized?

Logarithmic Differentiation:

From last time, we have the equation:

$$\frac{d}{dx}\left[\ln(u(x))\right] = \frac{u'(x)}{u(x)}.$$

We can rewrite this as:

$$u'(x) = \frac{d}{dx}\left[\ln(u(x))\right] u(x).$$

Sometimes it is easier to calculate the right hand side of this equation, than the left hand side directly.

Examples:

Differentiate:

(1) $f(x) = \frac{(3x+1)^{100}(x^2-5)^{10}}{\sqrt{3x+7}}$

(2) $f(x) = x^{15}e^{-3x+1}x^x(x^3 - 5x + 1)^\pi$

(3) $f(x) = \frac{e^{x^2}\sqrt{17x^3+x}}{(x+2)^{1000}}$