## The Antiderivative

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So, suppose that $f(x)$ is a function. A function $F(x)$ is called an antiderivative of $f(x)$ if the derivative of $F(x)$ is $f(x)$.

So, we have

$$F'(x) = f(x).$$

**Example:**

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$$F(x) = x^3 - x^2 + x.$$

(Another antiderivative is $F_1(x) = x^3 - x^2 + x + 7$.)

**Example:**

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- An antiderivative of $f(x) = e^{2x}$ is

$$F(x) = \frac{1}{2}e^{2x}.$$

(Another antiderivative is $\frac{1}{2}e^{2x} - 5$.)

(And another one is $\frac{1}{2}e^{2x} + 102$.)

## Fundamental Property of Antiderivatives

Suppose that $f(x)$ is a continuous function. If $F(x)$ and $G(x)$ are both antiderivatives of $f(x)$ then there is some constant $C$ so that

$$G(x) = F(x) + C.$$

## Fundamental Property of Antiderivatives

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$$G(x) = F(x) + C.$$

On the other hand, if $F(x)$ is an antiderivative of $f(x)$ then for any constant $C$ the function

$$G(x) = F(x) + C$$

is also an antiderivative of $f(x)$.

Therefore, if we can find **one** antiderivative of $f(x)$, then we know what **all** of the antiderivatives of $f(x)$ are.

## The Improper Integral

We have claimed that if $f(x)$ is a (continuous) function and $F(x)$ is an antiderivative of $f(x)$ then all antiderivatives of $f(x)$ take the form $F(x) + C$, for some constant $C$. We call this general antiderivative the **improper integral of $f(x)$** and we write

$$\int f(x)dx = F(x) + C.$$

The symbol '$\int$' is the **integral symbol** It's a stretched out $s$.

The **integrand** is the function being **integrated**, which is $f(x)$. The symbol '$dx$' means that the variable is $x$. The constant $C$ is the **constant of integration**.

[**WARNING:** When doing an improper integral, you should **always** write a constant of integration. Otherwise you **will** lose points on an exam.]

**Summary:**

The expression:

$$\int f(x)dx$$

represents an **arbitrary** antiderivative of $f(x)$. If $F(x)$ is any antiderivative of $f(x)$ then we write

$$\int f(x)dx = F(x) + C.$$

## Examples:

Compute the following improper integrals. (Don't forget the constant of integration!)

$$\int 7dx$$

$$\int 3x - 2dx$$

$$\int x^2 + 2x - 5dx$$

$$\int 2xe^{x^2}dx$$