Rules for integration (revisited and continued)

- The **constant rule**:

$$\int k dx = kx + C.$$

- The **power rule**: So long as $n \neq -1$,

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C.$$

- The **logarithm rule**: For $x \neq 0$,

$$\int \frac{1}{x} dx = \ln |x| + C.$$

- The **exponential rule**:

$$\int e^{kx}dx = \frac{1}{k}e^{kx} + C.$$

- The **constant multiple rule**: If $F(x)$ is an antiderivative of $f(x)$ then for any k

$$\begin{aligned}\int kf(x)dx &= k\int f(x)dx \\ &= kF(x) + C.\end{aligned}$$

- The **sum rule**. Suppose that $F(x)$ is an antiderivative of $f(x)$ and $G(x)$ is an antiderivative of $g(x)$. Then

$$\begin{aligned}\int f(x) + g(x)dx &= \int f(x)dx + \int g(x)dx \\ &= F(x) + G(x) + C.\end{aligned}$$

• The **difference rule**: Suppose that $F(x)$ is an antiderivative of $f(x)$ and $G(x)$ is an antiderivative of $g(x)$. Then

$$\begin{aligned}\int f(x) - g(x)dx &= \int f(x)dx - \int g(x)dx \\ &= F(x) - G(x) + C.\end{aligned}$$

Examples: Calculate the following integrals.

$$\int x^2 - 3x + 1 dx$$

$$\int \frac{3}{x} dx$$

$$\int y^{4.3} - 3y^{0.05} + \frac{1}{\sqrt{y}} dy$$

$$\int e^{3x} + x^{1.3} dx$$

$$\int \frac{3v^2 - 2v - 1}{v^2} dv$$

$$\int u^2 + \frac{1}{u} du$$

$$\int \sqrt{x}(x^2 + \frac{1}{x}) dx$$

$$\int (e^t + e^{-t})^2 dt$$

Calculating position and velocity from acceleration and initial data.

We know that velocity is the derivative of position, and that acceleration is the derivative of velocity. Therefore, if we know acceleration, we can find velocity and then position, by integrating twice. When we do this, we end up with **two different** constants of integration. (If we know initial position and velocity, then we can find the values of these constants of integration.)

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Example

Suppose a ball is catapulted in the air from the ground ($d = 0$) with an initial velocity of 49m/s. Gravity acts by acceleration at -9.8m/s^2 (the minus sign is to pull it towards the ground. How long does the ball stay in the air, and what is its maximum height?