Applications of the Definite Integral

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Suppose we want to find the area between two curves $f(x)$ and $g(x)$, on an interval $a \leq x \leq b$. [Suppose $f(x) \geq g(x)$.]

Then the **area** between these curves is

$$\int_a^b f(x) - g(x) dx$$

[Intuitively, this is the area beneath $f(x)$ minus the area beneath $g(x)$.]

In most examples of finding areas between curves, there are three steps:

(1) Find the interval $a \leq x \leq b$. (We'll see this in examples.) This involves finding the points where $f(x) = g(x)$.

(2) Set up the right definite integral.

(3) Correctly calculate the integral.

Examples:

(1) Find the area between the curves $f(x) = x$ and $g(x) = x^4$.

(2) Find the area between the curves $f(x) = 3 - x$ and $g(x) = x^3 - 3x^2 - x + 3$.

(3) Find the area of the region(s) between the graphs of $y = 4x - 4$ and $y = x^3 - 2x^2 - x + 2$.

[The book has a discussion of a topic called ‘Net Excess Profit’. I’m skipping this, and it won’t be on midterms or finals.]

Lorenz curves, the Gini Index and inequality.

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The value $L(x)$ is the y so that the bottom $100x\%$ have $100y\%$ of the income (or wealth). Let's do income.

For example, suppose that the poorest 50% of the population receives 30% of the total income. Then $L(0.5) = 0.3$.

If the poorest 99% of the population receive 80% of the income, then $L(0.99) = 0.8$.

Note that $L(0) = 0$ and $L(1) = 1$.

Also, if everyone receives exactly the same amount of income, then $L(x) = x$ for all x. Otherwise, we'll have $L(x) \leq x$.

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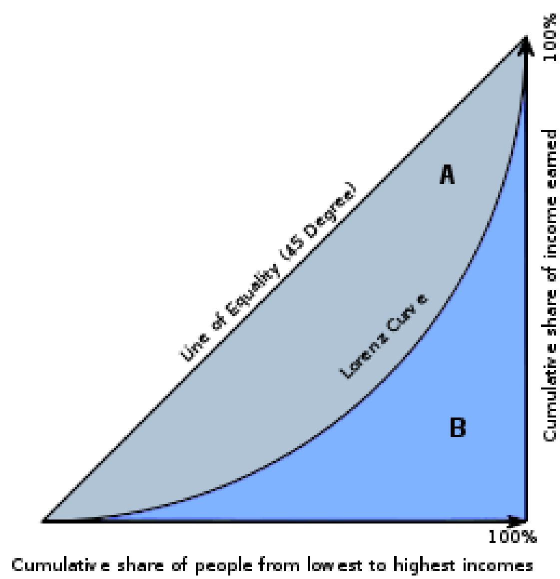
Typically, the Lorenz curve looks something like:



The *Gini Index* is twice the area between the line of inequality $y = x$ and the Lorenz curve $y = L(x)$:

$$\text{GI} = 2\int_0^1 [x - L(x)]dx.$$

(This is twice the grey area in the previous picture.)

The Gini Index is always between 0 and 1. A smaller number means that there is less inequality in income distribution.

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For example, the Gini Index of the U.S. is about 0.46, which is about the same as China, slightly less than Mexico, much less than Brazil, and quite a lot more than most European countries and Canada.

Of course, the U.S. has about 6 or 7 times the GDP per capita of China, so having the same Gini Index doesn't necessarily mean much...

The Gini Index is a measure of *relative inequality (of income)* within a particular population.

However, it is perhaps interesting that in 1967, the U.S. had a Gini index of about .34 and in 1991 it was .41 (and it seems to be going up and up...).

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Examples:

Calculate the Gini index for the following Lorenz functions:

(1) $L(x) = x^4$

(2) $L(x) = \frac{1}{3}x^3 + \frac{2}{3}x^5$.