The exponential function

Suppose that b is a real number, and n is a positive integer. Then

$b^n = b \cdot b \cdot \ldots \cdot b$

where b appears n times on the right.

If n is a positive integer then $b^{\frac{1}{n}} = \sqrt[n]{b}$.

Also, $b^{-n} = \frac{1}{b^n}$ and

$b^0 = 1$.

We'd like to be able to raise any number $b > 0$ to any other number x:

If b is a positive number other than 1 there is a unique function called the exponential function with base b that is defined by

$$f(x) = b^x$$

for any number x.

GRAPH A FEW OF THESE...

Properties of the exponential function:

If $b > 0$ and $b \neq 1$ then the exponential function $f(x) = b^x$ satisfies:

• It is defined, continuous and positive for all x.

• The x-axis is a horizontal asymptote.

• The y-intercept is $(0, 1)$ and there is no x-intercept.

• If $b > 1$ then $\lim\limits_{b \longrightarrow -\infty} b^x = 0$ and $\lim\limits_{b \longrightarrow \infty} b^x = \infty$.

If $0 < b < 1$ then $\lim\limits_{b \longrightarrow -\infty} b^x = \infty$ and $\lim\limits_{b \longrightarrow \infty} = 0$.

• If $b > 1$ then the graph is decreasing for all x. If $0 < b < 1$ then the graph is decreasing for all x.

NOTE: The function $f(x) = 2^x$ is very different from the function x^2. We'll do the derivative later, but you CANNOT apply the power rule...

RULES of the exponential function: (see p. 296 of the text)

Equality rule: $b^x = b^y$ if and only if $x = y$.

Product rule: $b^x b^y = b^{x+y}$

Quotient rule: $\frac{b^x}{b^y} = b^{x-y}$

Power rule: $(b^x)^y = b^{xy}$

Multiplication rule: $(ab)^x = a^x b^x$

Division rule: $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

EXAMPLES

The number e

For the exponential function, there is a special number e. There are lots of ways to define e, and its value is approximately:

$$2.7182818284...$$

It is the unique number e so that the slope of the function

$$f(x) = e^x$$

at $x = 0$ is 1.

It also true that:

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

For reasons we're not really going to go into (except one important concept on Friday), the number e, along with the function

$$f(x) = e^x$$

are two of the most important concepts in mathematics...

Things to do with the exponential functions:

• Practice the rules of manipulating them.

• Use your calculator to calculate different values of the function.

• Have them as more functions which are defined everywhere, continuous everywhere (and actually have derivatives everywhere, and so on). So we get lots more examples of functions to differentiate.

Next time, we'll see that the exponential function can be used to calculate interest (and other things) when compounding happens **continuously**.