Functions of several variables

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Unless stated otherwise, the domain is the largest set of pairs (x, y) where the function is defined.

[So, no taking square roots of a negative number, and no dividing by 0. And no taking ln(x), where $x \leq 0$...]

Examples:

What are the domains of the following functions?

(1) $f(x, y) = x^2 + y^2$

(2) $f(x, y) = \sqrt{1 - x^2 - y^2}$

(3) $f(x, y) = \frac{x^2 - y}{x + 2y}$.

Note that some of the functions we've already seen can be treated as functions of several variables:

$$B(r, t, P) = Pe^{rt}$$

We can consider the amount made from an investment as a function of the principal, the rate and the time (rather than just of the time).

The graph of a function $f(x, y)$ happens in 3-dimensional space.

We have the xy-plane (horizontal), and add another direction, up/down. Then, a point has three co-ordinates (x, y, z). This lies directly above (if z is positive, below if z is negative) the point (x, y) in the xy-plane.

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The the graph consists of the collection of points $(x, y, f(x, y))$, where (x, y) is in the domain of $f(x, y)$.

[EXAMPLE graphs]

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Fix a value k, and draw the curve $f(x, y) = k$. Do this for different values of k. If we label these curves with the value of k, we get a reasonable understanding of the graph of the function $z = f(x, y)$

This is like a map with contour lines showing the heights.

[EXAMPLE pictures of level curves.]

Lines of indifference (and isoquants)

In economics, suppose we have a product that is produced with two inputs (like the raw materials and the labor).

$$Q(x, y)$$

Isoquants

In economics, suppose we have a product that is produced with two inputs (like capital investment and labor (hours)).

$$Q(x, y)$$

If we fix a value C of production, then we get a curve $Q(x, y) = C$. This is called the curve of constant product C, also known as an isoquant.

[If you increase capital investment, you may be able to decrease the hours of labor while keeping production constant.

This seems to be a theme of modern business...]

Indifference curves

Suppose that a consumer is considering buying some amount of one good and some of another. There is an associated **utility function** $U(x, y)$ which measures the total satisfaction (or utility) the consumer derives from having x units of the first good, and y of the second.

Indifference curve

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A level curve $U(x, y) = C$ is called a **indifference curve**. It gives the different ways that combinations of the two goods can be put together to give the level of satisfaction C.

Example

Suppose that the utility derived from x units of good A and y units of good B is

$$U(x, y) = x^2y + xy^2 + x$$

(1) If the consumer currently has 10 of good A and 12 of good B, what is their current utility?

(2) Sketch the indifference curve associated to this level of utility.