From last time:

For a population (and their income), the Lorenz curve is the map defined for $0 \leq x \leq 1$ as follows:

If the bottom $100x\%$ of the population receive $100y\%$ of the total income of the population, then $L(x) = y$.

We can easily see that $L(0) = 0$ and $L(1) = 1$, and that for any other x we have $L(x) \leq x$.

The Gini Index is twice the area between the line of total equality $y = x$ and the curve $L(x)$. So

$$\text{GI} = 2 \int_0^1 [x - L(x)]\, dx.$$

Last time, we left off with wanting to do the following examples:

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Examples:

Calculate the Gini index for the following Lorenz functions:

(1) $L(x) = x^4$

(2) $L(x) = \frac{1}{3}x^3 + \frac{2}{3}x^5$.

The Average Value of a Function

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For example, the average temperature for a day, or the average speed of a trip.

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The Average Value of a function: Suppose that $f(x)$ is a function that is continuous on the interval $a \leq x \leq b$. Then the **average value** V of $f(x)$ over $a \leq x \leq b$ is given by the definite integral

$$V = \frac{1}{b-a} \int_a^b f(x)dx.$$

You should think that

$$\int_a^b f(x)dx$$

is the cumulative total of $f(x)$, over the interval $a \leq x \leq b$. So, we're taking the total amount, and dividing by the length of the interval.

For example, if we're thinking about average speed (and velocity is positive), then

$$\int_a^b f(x)dx$$

is the distance travelled, so when we divide by $b - a$ we get the average speed.

Examples:

(1) What is the average value of the function $f(x) = 3x + 1$ over the interval $0 \leq x \leq 2$?

(2) What is the average value of the function $f(x) = x^2 - 3x + 1$ over the interval $1 \leq x \leq 3$?

(3) What is the average value of the function $f(x) = \frac{2x}{(x^2-1)^2}$ over the interval $2 \leq x \leq 5$?

Business/Economics Applications of the Integral

[1] Useful life of equipment

Suppose that we have a piece of equipment which produces revenue with a function $R(t)$ (say t is in years) and cost function $C(t)$. The profit generated is

$$P(t) = R(t) - C(t).$$

Profit will be increasing when $R'(t) > C'(t)$, and decreasing when $R'(t) < C'(t)$.

Business/Economics Applications of the Integral

[1] Useful life of equipment

Suppose that we have a piece of equipment which produces revenue so that after t years the **total** revenue produced has been $R(t)$ (say t is in years) and the total cost of running the machine for y years has been $C(t)$. The total profit generated for the first t years is

$$P(t) = R(t) - C(t).$$

Profit will be increasing when $R'(t) > C'(t)$, and decreasing when $R'(t) < C'(t)$.

Since old machines cost more and produce less (since they are running less efficiently) we can expect that at a certain point

(time T) we will change from increasing profit to decreasing profit (i.e. we'll have $R'(T) = C'(T)$).

At this point, the total profit generated by the machine is maximized. This is considered a good time to replace the machine, and the time period $0 \leq t \leq T$ is called the useful life of the machine.

The net profit over the useful life of the machine gives an indication of the value of the machine.

Of course, then we can do more sophisticated things like compare the cost of a new machine with the extra profit it will generate, in order to see how long the investment in a new machine would take to pay off, and when the best time to get a new machine might be (depending on the situation, it could be before, at, or after the end of the useful life of the machine.

Example:

Suppose that after t years the cost of a machine (in thousands of dollars) is

$$C'(t) = 100 + 20t,$$

whereas the after t years the revenue is being generated at a rate of

$$R'(t) = 300 - 40t.$$

(1) What is the useful life of the machine?

(2) What is the net profit over the useful life of the machine?

Future and present value of an income flow

[Left until next time.]