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[A **relative maximum** for $f(x, y)$ is a point (a, b) so that for some small circular disk P centered at (a, b) and for all (x, y) in P we have

$$f(x, y) \leq f(a, b).$$

And relative minimum is defined similarly ...]

Definition: Suppose that (a, b) is in the domain of $f(x, y)$ and suppose that f_x and f_y both exist at (a, b).

We say that (a, b) is a **critical point for f** if

$$f_x(a, b) = 0$$

and

$$f_y(a, b) = 0.$$

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[PICTURE]

The Second Partial Derivative Test for relative extrema.

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Remember: $D = f_{xx}f_{yy} - f_{xy}^2$.

Here's the test:

For a critical point (a, b) of $f(x, y)$, consider the number $D(a, b)$.

- If $D(a, b) < 0$ then (a, b) is a **saddle point** of $f(x, y)$.

- If $D(a, b) > 0$, consider $f_{xx}(a, b)$.

If $f_{xx}(a, b) > 0$ then (a, b) is a **relative minimum** for $f(x, y)$.

If $f_{xx}(a, b) < 0$ then (a, b) is a **relative maximum** for $f(x, y)$.

[Finally, if $D(a, b) = 0$ then the test is inconclusive.]

Examples:

First, let's verify (using the above test) that the function $f(x, y) = x^2 - y^2$ really does have a saddle point at $(0, 0)$.

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Now, let's find the critical points, and classify them, for the following functions.

$$
\begin{aligned}
f(x,y) &= x^2y^2 \\
f(x,y) &= x^2 + y^2 - 4x + 4y + 6 \\
f(x,y) &= xy(x-2)(y+3)
\end{aligned}
$$