There is **another** online survey for those of you (freshman) who took the ALEKS placement test before the semester. Please follow the link at the Math 165 web-page, or just go to:

https://illinois.edu/sb/sec/2457922

The final exam will be held on

Wednesday, December 7 6-8pm in BSB 250.

[This information is posted on the Exams section of the 165 website.]

The final exam will be held on

Wednesday, December 7 6-8pm in BSB 250.

[This information is posted on the Exams section of the 165 website.]

There are also some old exams and some practice questions there.

Lagrange Multipliers

Lagrange multipliers are used to maximize or minimize a function whose domain is *constrained* by a different equation.

What we're going to do today is explore what this means, and then what the method is, and run some examples.

Setup:

We have a function $f(x,y)$ and we're considering points (x,y) so that $g(x,y)=0$ (the 'constraint').

So, let

$$C = \{(x,y) \mid g(x,y) = 0$$

be the set of points satisfying the constraint.

We'd like to find the maximum and minimum values of $f(x,y)$ for points $(x,y) \in C$.

Example:

Example:

Suppose that a plant needs to be watered. So, you need to go down to the river to fill the bucket, and then go to the plant. We're on a flat field. How should we travel?

Suppose that the river follow a curve $g(x, y) = 0$, that we start at a point M and that the plant is at a point C.

Example:

Suppose that a plant needs to be watered. So, you need to go down to the river to fill the bucket, and then go to the plant. We're on a flat field. How should we travel?

Suppose that the river follow a curve $g(x, y) = 0$, that we start at a point M and that the plant is at a point C.

Our best route is to go straight to the river, to some point $P = (x, y)$, and from there straight to C.

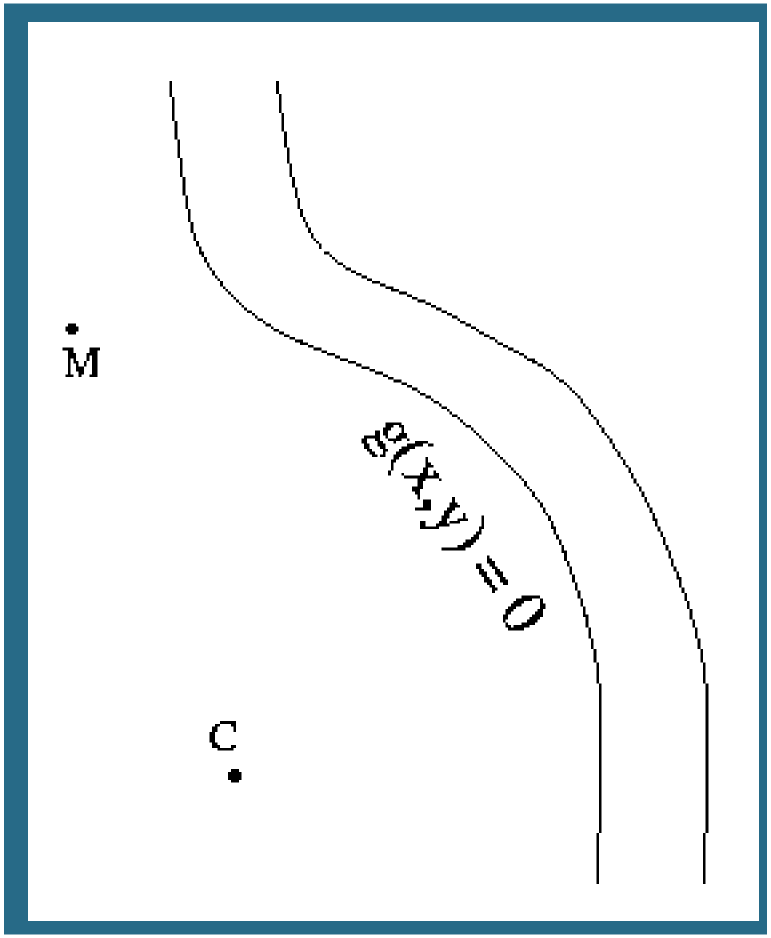
Example:

Suppose that a plant needs to be watered. So, you need to go down to the river to fill the bucket, and then go to the plant. We're on a flat field. How should we travel?

Suppose that the river follow a curve $g(x, y) = 0$, that we start at a point M and that the plant is at a point C.

Our best route is to go straight to the river, to some point $P = (x, y)$, and from there straight to C. But which point P?

Here's a picture:


M
g(x,y) = 0
C

So, we want to **minimize** the function

$$f(x, y) = d(M, (x, y)) + d((x, y), C)$$

(d is distance, and $P = (x, y)$)

subject to the constraint that P lies on the river, so $g(x, y) = 0$.

This is what Lagrange multipliers are for.

Second example:

Second example:

Suppose we have a closed bounded domain D (say a disk) and we want to maximize and minimize the function $f(x, y)$ on the domain D.

Then, we first find critical points in the interior of D (as we did last time).

Second example:

Suppose we have a closed bounded domain D (say a disk) and we want to maximize and minimize the function $f(x, y)$ on the domain D.

Then, we first find critical points in the interior of D (as we did last time).

Then we need to 'check the endpoints', which means maximize and minimize the function f on the 'boundary' of D – and this is where the constraint comes in.

Second example:

Suppose we have a closed bounded domain D (say a disk) and we want to maximize and minimize the function $f(x, y)$ on the domain D.

Then, we first find critical points in the interior of D (as we did last time).

Then we need to 'check the endpoints', which means maximize and minimize the function f on the 'boundary' of D – and this is where the constraint comes in.

OK, so let's go on to the method.

Method:

Method:

• Suppose we're trying to maximize and minimize $f(x, y)$ subject to the constraint $g(x, y) = 0$.

Method:

• Suppose we're trying to maximize and minimize $f(x, y)$ subject to the constraint $g(x, y) = 0$.

(1) First calculate the partial derivatives f_x and f_y.

Method:

• Suppose we're trying to maximize and minimize $f(x, y)$ subject to the constraint $g(x, y) = 0$.

(1) First calculate the partial derivatives f_x and f_y.

(2) The maximum/minimum values of f subject to the constraint occur at points (a, b) where there is a number λ (the Lagrange multiplier) so that

$$\begin{aligned} f_x(a,b) &= \lambda g_x(a,b) \\ f_y(a,b) &= \lambda g_y(a,b) \text{ and} \\ g(x,y) &= 0. \end{aligned}$$

(3) Find all such a, b, λ, and then plug in the values $f(a, b)$ to see which are the biggest.

Examples:

Examples:

Find the maximum and minimum values of the function

$$f(x, y) = x + 2y$$

subject to the constraint

$$3x^2 + 4y^2 - 3 = 0.$$

Examples:

Find the maximum and minimum values of the function

$$f(x, y) = x + 2y$$

subject to the constraint

$$3x^2 + 4y^2 - 3 = 0.$$

Find the maximum and minimum values of the function

$$f(x, y) = 3x^2 + 2y^2 + 2,$$

subject to the constraint

$$x^2 + 4y^2 - 4 = 0.$$