Review continued from last time...

Elasticity of demand

Suppose that p is the price of a commodity and the demand q satisfies

$$q = D(P),$$

for some nice function D. The **price elasticity of demand** is

$$E(p) = \frac{p}{q}\,\frac{dq}{dp}.$$

$E(p)$ is the percentage rate of change in q produced by a 1% rate of change in price p.

If $|E(p)| > 1$ then demand is **elastic**.

If $|E(p) < 1$ then demand is **inelastic**.

If $|E(p)| = 1$ then demand is of **unit elasticity**.

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Revenue is $R = pq = pD(p)$.

If demand is elastic then as price increases revenue decreases.

If demand is inelastic then as price increases revenue increases.

If demand is of unit elasticity then revenue is unchanged by a small change in price.

Example:

Suppose that demand is given by

$$D(p) = \sqrt{400 - 0.01p^2}$$

and that the price is $p = 120$. What is the elasticity of demand?

Compound interest

If we invest P dollars are invested at an interest rate r and interest is compounded k times per year then after t years the accumulated value is

$$B(t) = P\left(1 + \frac{r}{k}\right)^{kt}.$$

If interest is compounded continuously then the accumulated value is

$$B(t) = Pe^{rt}.$$

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Exponential models

The quantity $Q(t)$ has **exponential growth** if

$$Q(t) = Q_0 e^{kt}$$

for some $k > 0$. [Q_0 is the initial quantity.]

It has **exponential decay** if (for $k > 0$) we have

$$Q(t) = Q_0 = e^{-kt}.$$

The Learning Curve

If A, B and k positive constants the **learning curve** is

$$Q(t) = B - Ae^{-kt}$$

At $t = 0$ we have $Q(0) = B - A$, and there is a horizontal asymptote at B.

[Meaning

$$\lim_{t \to \infty} Q(t) = B. \quad]$$

The Logistic Curve

Again, A, B and k are positive constants. The **logistic curve** is

$$Q(t) = \frac{B}{1 + A^{-Bkt}}$$

This curve starts at $Q(0) = \frac{B}{1+A}$ and has a horizontal asymptote at B.

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This last equation can be turned into

$$f'(x) = \frac{d}{dx}\left[\ln(f(x))\right].f(x)$$

and used for logarithmic differentiation.

Integration

An antiderivative of a function $f(x)$ is a function $F(x)$ so that

$$F'(x) = f(x).$$

If $G(x)$ is any other antiderivative of $f(x)$ then there is a constant C so that

$$G(x) = F(x) + C$$

The notation for the antiderivative of $f(x)$ is

$$\int f(x)dx = F(x) + C,$$

and is called the **indefinite integral of $f(x)$**.

We had a bunch of rules:

- Constant multiple rule
- Power rule
- Sum rule
- Logarithmic rule:

$$\int \ln(x)dx = \frac{1}{x} + C$$

- Exponential Rule:

$$\int e^{kx}dx = \frac{1}{k}e^{kx} + C$$

WARNING: There is no product rule or quotient rule for integrals.

Integration by substitution

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Then

$$\int f(x)dx = G(u(x)) + C.$$

Examples

Calculate the following indefinite integrals:

$$\int (x-2)(x^2-4x)^2 dx$$

$$\int (2x-1)e^{x2-x} dx$$

$$\int \frac{x+1}{x-1} dx$$

The Definite Integral

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The **Fundamental Theorem of Calculus** says that if $F(x)$ is an antiderivative of $f(x)$ and $f(x)$ is continuous on the interval $a \le x \le b$ then

$$\int_a^b f(x)dx = F(b) - F(a).$$

There are rules for definite integrals too:

Constant multiple rule, sum rule, difference rule, and also:

$$\int_a^a f(x)dx = 0$$

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

Area between curves

Suppose that $f(x)$ and $g(x)$ are functions and that for $a \leq x \leq b$ we have $f(x) \geq g(x)$.

The **area between the curves $y = f(x)$ and $y = g(x)$ over the interval $a \leq x \leq b$** is

$$A = \int_a^b f(x) - g(x) dx.$$

Average value

Suppose that $f(x)$ is a function which is continuous on the interval $a \leq x \leq b$. The **average value** of $f(x)$ over the interval $a \leq x \leq b$ is

$$Av. = \frac{1}{b-a}\int_a^b f(x)dx.$$

Applications

Here are some of the applications of the integral which we saw:

- Lorenz curves
- Gini index.
- Useful life of equipment.
- Future and present values of an income flow.
- Consumer willingness to spend.
- Consumers' and Producers' Surplus.

Functions of two variables

- Domains
- Partial Derivatives

[NOTE: Lagrange Multipliers will **not** be on the final.]