Math 165: Calculus for Business

http://www.math.uic.edu/coursepages/math165/index_html

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Difference Quotients.

In this example, f is a function, x is the variable, and h is some number (not 0). The difference quotient is then

$$\frac{f(x+h)-f(x)}{h}.$$

[This is an important concept in calculus, but it's also a good illustration of functions, and composite functions.]

Example 1

Let $f(x) = 3x$. Then the difference quotient is

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{3(x+h)-3x}{h} \\ &= \frac{3h}{h} \\ &= 3.\end{aligned}$$

Example 2

Let $f(x) = x^2 + 7$. Then the difference quotient is

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{(x+h)^2 + 7 - \left(x^2 + 7\right)}{h} \\ &= \frac{x^2 + 2xh + h^2 + 7 - x^2 - 7}{h} \\ &= \frac{2xh + h^2}{h} \\ &= 2x + h.\end{aligned}$$

The Vertical Line Test.

When we graph a function, the graph meets any vertical line in the plane in at most one point.

[So, some curves are not the graph of a function. Like a circle.]

Linear functions

A linear function is a function f where there are **constants** m and c so that

$$f(x) = mx + c.$$

'slope-intercept form'

Examples

$$
\begin{aligned}
f_1(x) &= 3x + 5 \\
f_2(x) &= x \\
f_3(x) &= (\sqrt{2})x - 8 \\
f_4(x) &= 2x + \pi \\
f_5(x) &= 7.
\end{aligned}
$$

[Graph these functions.]

A linear function changes in value at a constant rate with respect to the variable.

If $f(x) = mx + c$ then the **slope** of f is m. We can see that c is the y-intercept. The slope measures how steep the graph is.

If we have x_1 and x_2 in the domain, then we can also see that:

$$m = \frac{f(x_1) - f(x_2)}{x_1 - x_2}.$$

OR, using difference quotients, we see that for the linear function $f(x) = mx + b$ we have

$$\frac{f(x+h) - f(x)}{h} = m.$$

Point-slope form

If we have $f(x_0) = y_0$ and slope m then we can write the points (x, y) for $y = f(x)$ as those points which satisfy:

$$y - y_0 = m(x - x_0).$$

This is the point-slope form.

You also see $y = m(x - x_0) + y_0$, which is written more like a function.