Notice:

For freshman who took the ALEKS Placement test, could you please fill out a survey at:

`https://illinois.edu/sb/sec/5271145`

There's a link on the Math 165 website.

Properties of limits (see page 66):

Limits do not always need to exist. However:

If $\lim_{x \to c} f(x)$ and $\lim_{x \to c} g(x)$ both exist then:

- $\lim_{x \to c} [f(x) + g(x)] = \lim_{x \to c} f(x) + \lim_{x \to c} g(x)$
- $\lim_{x \to c} [f(x) - g(x)] = \lim_{x \to c} f(x) - \lim_{x \to c} g(x)$
- For any constant k we have $\lim_{x \to c} kf(x) = k \lim_{x \to c} f(x)$
- $\lim_{x \to c} f(x)g(x) = \left(\lim_{x \to c} f(x)\right)\left(\lim_{x \to c} g(x)\right)$

- If $\lim_{x \to c} g(x) \neq 0$ then $\lim_{x \to c} \left(\frac{f(x)}{g(x)} \right) = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}$

- If $\left(\lim_{x \to c} f(x) \right)^p$ exists then

$$\lim_{x \to c} f(x)^p = \left(\lim_{x \to c} f(x) \right)^p$$

A **polynomial** function has the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

for some numbers $a_0, \ldots, a_n$.

A **rational** function has the form

$$f(x) = \frac{p(x)}{q(x)}$$

where $p(x)$ and $q(x)$ are polynomials.

Examples of polynomial functions:

$f_1(x) = 7x^3 - 5x^2 - 9$

$f_2(x) = x^2 - 2x - 3$

Linear functions are polynomials.

Examples of rational functions:

$f_3(x) = \frac{1}{x+1}$

$f_4(x) = \frac{x-1}{x^2+x+1}$

If $p(x)$ and $q(x)$ are polynomials, and $q(c) \neq 0$ then:

- $\lim_{x \to c} p(x) = p(c)$

- $\lim_{x \to c} \frac{p(x)}{q(x)} = \frac{p(c)}{q(c)}$

Examples...

Limits at infinity

We say that

$$\lim_{x \to \infty} f(x) = L$$

if when x gets larger and larger the value of $f(x)$ gets closer and closer to L.

(Similar definition for $\lim_{x \to -\infty} f(x) = L'$...)

Examples

- $\lim_{x \to \infty} \frac{1}{x} = 0$

- $\lim_{x \to \infty} \frac{2x^3 - 3x^2 + 5x - 100}{x^3 + 3x + 1} = 2.$

We write

$$\lim_{x \to c} f(x) = \infty$$

if $f(x)$ gets larger and larger as x approaches c. Also, if $f(x)$ decreases more and more as x goes to c we write

$$\lim_{x \to c} f(x) = -\infty.$$

Example

$\lim_{x \to 0} \frac{1}{x^2} = \infty.$

(But not for $\frac{1}{x}$...)

One-sided limits:

A one-sided limit is where we only consider values of the function to one-side (the left or the right) of the value c. So if we're only consider values of x less than c (but close to c) we write

$$\lim_{x \to c^-} f(x) = L,$$

whereas if we're considering values from above ($x > c$), we write

$$\lim_{x \to c^+} f(x) = L.$$

Example

Consider $\lim_{x \to 0} \sqrt{x}$. Well, the square root is not defined for negative values, so we can't look at *all* values close to 0. On the other hand:

$$\lim_{x \to 0^+} \sqrt{x} = 0.$$

The ordinary (two-sided) limit exists if and only if both one-sided limits exist and are equal (and this is then the value of the ordinary limit...).

Continuity

A function $f(x)$ is continuous at c if

$$\lim_{x \to c} f(x) = f(c).$$

(Namely, f is defined at c, the limit exists at c and the limit is equal to $f(c)$.)

Polynomial functions are continuous everywhere.

Rational functions are continuous everywhere they're defined.