Marginal Cost

Suppose that we have a production function $C(x)$, which measures the cost of producing x units (of something).

The **marginal cost** of producing x_0 units is the derivative, $C'(x_0)$.

This is an approximation to the cost of producing 1 more unit when production is x_0:

$$C(x_0 + 1) - C(x_0).$$

Similarly, suppose the revenue function is $R(x)$, and the profit function is $P(x)$.

Then the **Marginal Revenue** at production x_0 is $R'(x_0)$, and

the **Marginal Profit** is $P'(x_0)$.

Example

Suppose that a manufacturer finds that they have the following cost function:

Cost: $C(x) = 2,000 + 800\sqrt{x}$.

The price function is constant, with $p(x) = 100$.

What is the revenue function? What is the profit function? What is the break-even point?

What is the marginal cost at $x = 144$? What is the cost of producing one more unit when 144 units are produced?

What are the marginal revenue and marginal profit at $x = 144$?

Approximation by increments

The derivative can be used to estimate the change in a function for a small change in x. Since

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

for small values of h we have that $f'(x)$ is almost the difference quotient.

Then we get that

$$f(x_0 + h) - f(x_0) \approx f'(x_0).h$$

('$\approx$' means 'approximately equal to')

We often use Δ for change. We can rewrite the previous equation as

$$\Delta f \approx f'(x_0)\Delta x.$$

(Careful with this, Δf doesn't seem to involve x_0, but it means

$$f(x_0 + \Delta x) - f(x_0),$$

and we've written Δx for h...)

[By the way, all of these are different ways of talking about the equation of the tangent line to the graph of $f(x)$ at $x = x_0$...]

Approximate percentage rate of change

The percentage change in f, at value x_0, with change Δx is

$$100\frac{\Delta f}{f(x_0)}.$$

This is approximately

$$100\frac{f'(x_0)\Delta x}{f(x_0)}.$$

Example:

A stone is catapulted into the air, and the height (in meters) at time t (seconds), after being thrown is

$$h(t) = 50t - 5t^2.$$

1. What is the height at time $t = 3$?

2. Estimate the height at times $t = 3.01, 3.1$ and 4.

3. How close were we to the right answer?

Example:

The cost (in thousands of dollars) of manufacturing x thousand gadgets is

$$C(x) = 500 + 20x - \frac{1}{5}x^2 + \frac{10}{50^3}x^4.$$

Currently, $50,000$ gadgets are being produced. (So $x = 50$.) Estimate how many we extra gadgets could produce if we were prepared to spend another 100 thousand dollars.